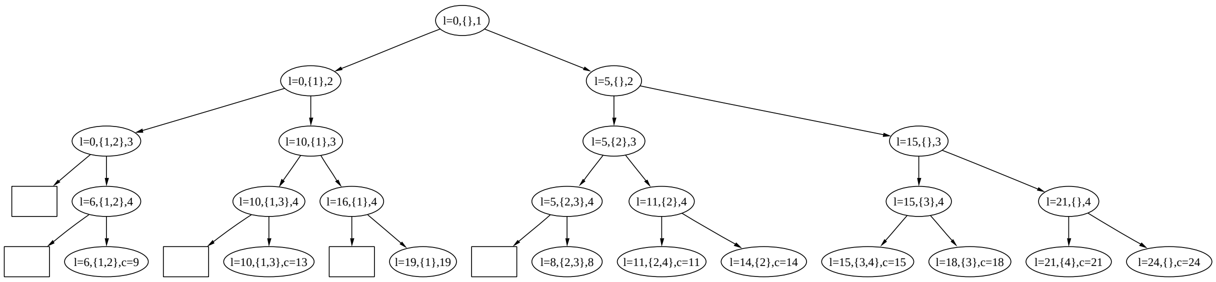
1. Job scheduling

The jobs are defined as a sequence of triples:

i	p_i	d_i	t_i
1	5	1	1
2	10	2	3
3	6	2	1
4	3	1	1



The l is the lower bound for each node, also called estimated cost:

$$l(X) = \sum_{q(x)} p_i, \text{ where } q(x) = i \notin S_x \wedge i < max(S_x)$$

The upper bound for each node is

$$u(X) = \sum_{q(x)} p_i, \text{ where } q(x) = i \notin S_x$$

2. PRAM search

Given an

$$X \in \mathbf{Int}$$

and

$$S.0..M-1 \in \mathbf{Int} \text{ where } \forall i,j : 0 \le i < j < M : S.i < S.j$$

one is required to calculate *present* such that

$$present \equiv \exists i : 0 \le i < M : X = S.i$$

Determining *present* is then relaxed to finding an i

$$R \equiv 0 \le i < M \wedge S.i \le X < S.i+1$$

assuming that $S.M$ is defined to be greater than X.

Dijkstra's program has the form

$|[\mathbf{Var}\ i \in \mathbf{Int}$
$\quad ;\ S.M := X+1$
$\quad ;\ establish\ R$
$\quad ;\ present := (S.i = X)$
$]|$

$$P \equiv 0 \le i < j \le M \wedge S.i \le X < S.j$$

$$P \textbf{ invariant}$$

A progress condition:

$$(j-i) > (j-i)'$$

A fixed point:

$$(j-i) = 1$$

"establish R" is

$|[\mathbf{Var}\ j \in \mathbf{Int}$
$\quad ;\ i,j := 0,M$
$\quad ;\ \{P\}$
$\quad *|[$
$\qquad j-i \ne 1 \to shrink(S,j,i)\{P\}$
$\quad]|\{R\}$
$]|\ \{R\}$

Shrinking $(j-i)$ can be done by either:

- increasing i to some h, if $S.h \leq X$;
- or decreasing j to h, if $S.h < X$;
- or both simultaneously, why not?

$$
\begin{array}{l}
shrink(S, X, i, j) \ \mathbf{is} \\
\quad |[\mathbf{Var}\ h \in \mathbf{Int} \\
\quad\ ;\ h := (i + j)/2 \\
\quad\ ;\ \mathbf{if}\ S.h \leq X \rightarrow i := h \\
\quad\quad [\!]\ X < S.h \rightarrow j := h \\
\quad\quad \mathbf{fi} \\
\quad\ ]|
\end{array}
$$

Program correctnes?

- P is true initially;
- P holds after every transition;
- $(j - i) > 0$ and decreases with every transition;
- $(j - i) = 1$ $X = S.i$ or $X < S.0$ and these conditions exclude each other.

3. (P+1)-ARY SEARCH

The program becomes

$$
\begin{array}{l}
|[\textbf{Var } i \in \textbf{Int} \\
\quad ;\ \textit{establish } R \\
\quad ;\ |[||\ i : i \le k < min(i+p, M) : \\
\qquad\quad q := (X = S.k) \\
\qquad]| \\
]|
\end{array}
$$

The loop invariant is the same. The fixed point now is reached when $(j-i)$ is less than or equal to p.

"establish R" is then refined to

$$
\begin{array}{l}
|[\textbf{Var } j \in \textbf{Int} \\
\quad ;\ i, j := 0, M \\
\quad ;\ \{P\} \\
\quad *|[\\
\qquad j - i > p \rightarrow pshrink(S, j, i, p)\{P\} \\
\qquad]|\{R\} \\
]|\ \{R\}
\end{array}
$$

This is performed by a parallel test.

$$
\begin{array}{l}
pshrink(S, X, i, j)\ \textbf{is} \\
\quad |[\textbf{Var } h \in \textbf{Int} \\
\qquad ;\ nel := (i+j)/(p+1) \\
\qquad ;\ |[||\ k : 1 \le k \le p : \\
\qquad\qquad \textbf{Var } h, hb \in \textbf{Int} \\
\qquad\qquad ;\ h, hb := i + (nel * k), i + (nel * (k-1)) \\
\qquad\qquad ;\ \textbf{if } k = p \wedge X \ge S.h \rightarrow i := h \\
\qquad\qquad\quad [\!]\ S.hb \le X \wedge X < S.h \rightarrow i, j := hb, h \\
\qquad\qquad\quad \textbf{fi} \\
\qquad]| \\
\quad]|
\end{array}
$$

4. Sequence sums

Given

$$s_i = 0, \text{ for } i \leq 0$$
$$a_{1..m}, b_{1..m}$$

calculate

$$s_i = (a_i * s_{i-1}) + b_i$$

s_i is reduced to:

$$\sum_{1 \leq i < n} ((\prod i < j \leq n a_j) * b_i)$$

; $scanl(a, 1, n, s)$
; $\|[\|$ $i : \leq\leq 1in :$
$\quad s.h := s.h * (b.i/a.i)$
$]\|$
; $scanl(s, 1, n, s)$;