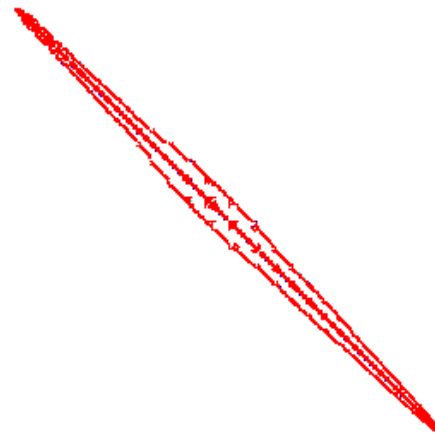
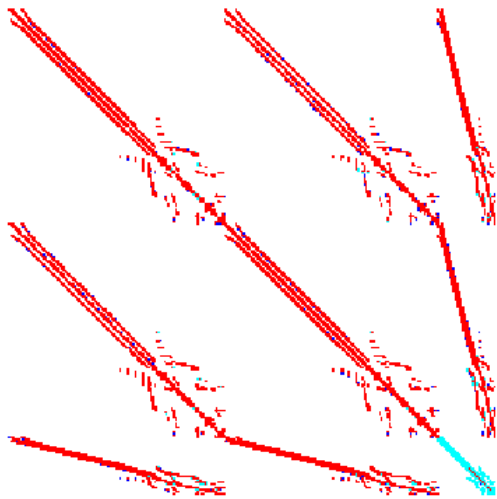
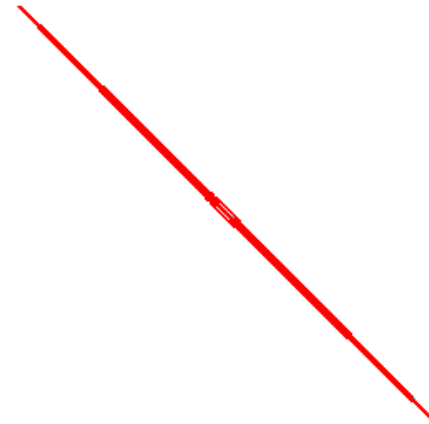
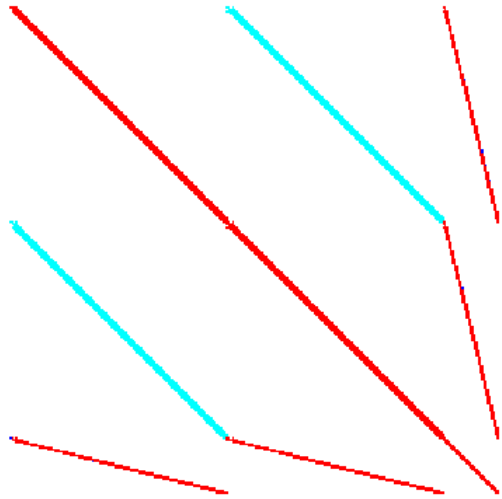


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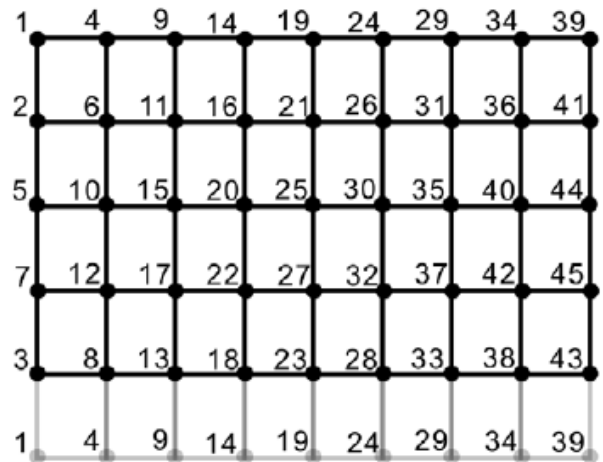
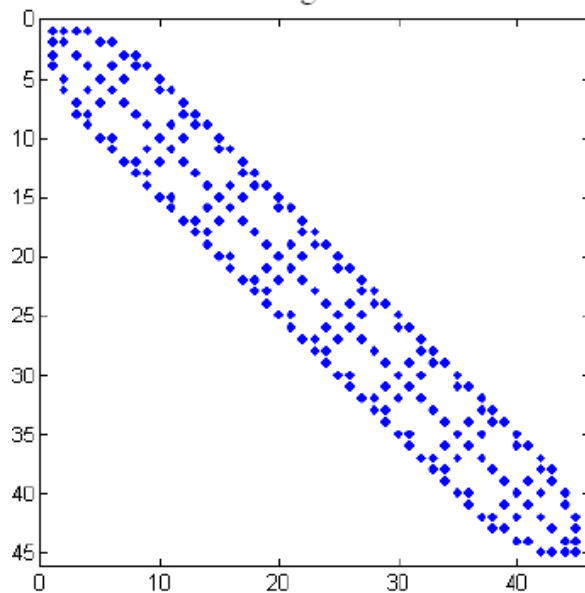
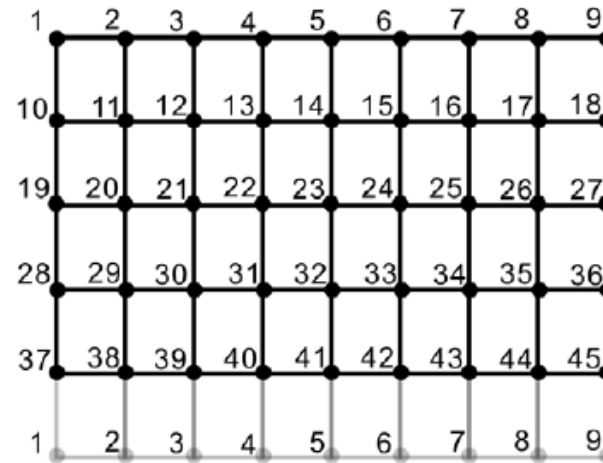
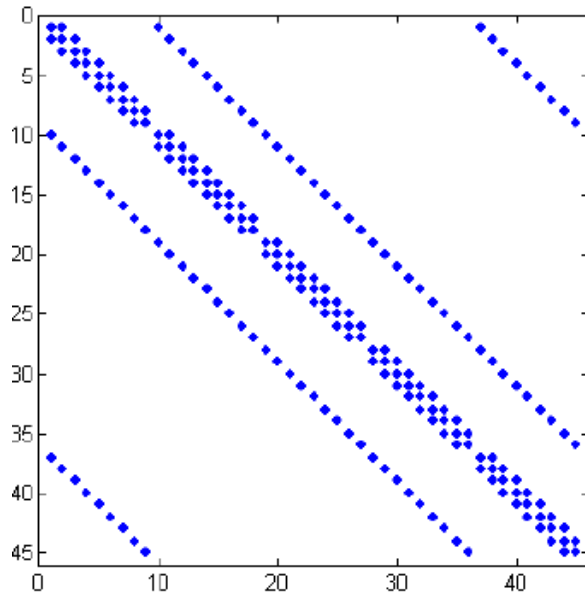
Armazenamento de Matrizes Esparsas

Lucia Catabriga

Matrizes Esparsas

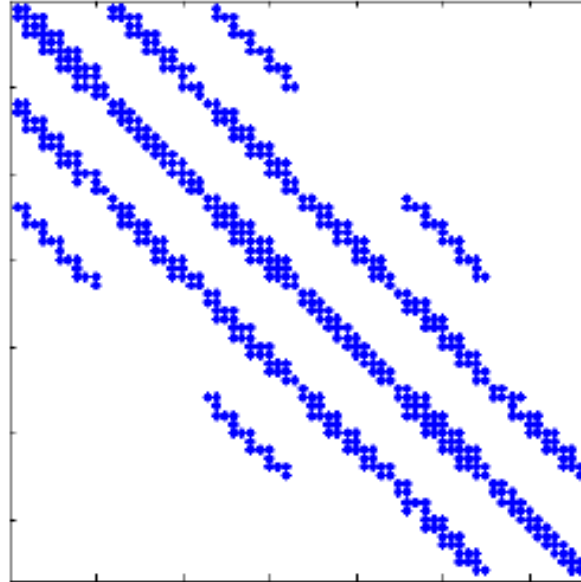


Matrizes Esparsas

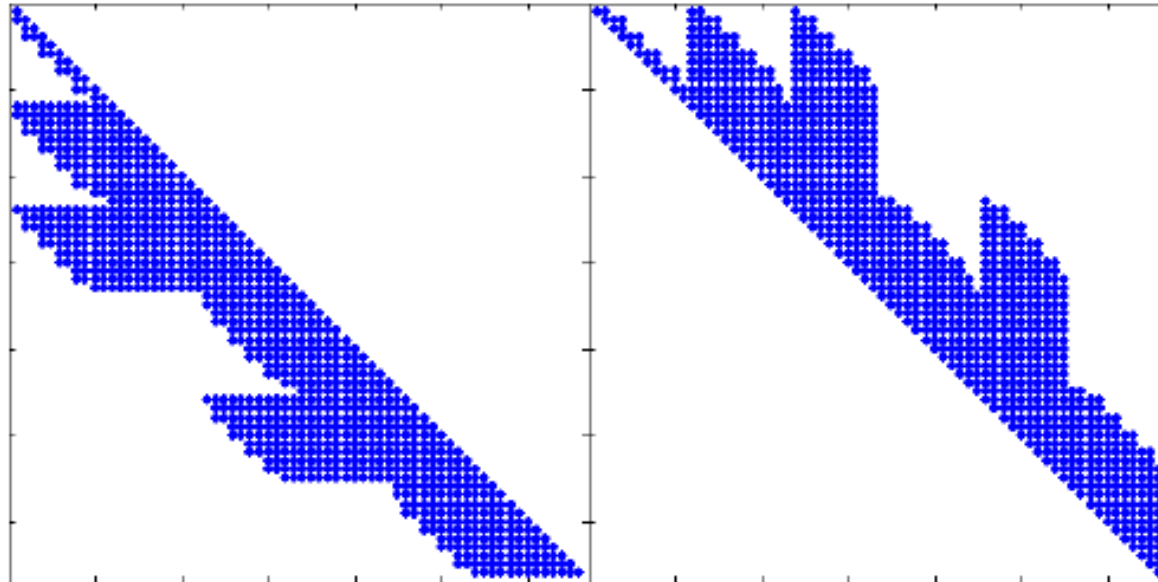


Matrizes Esparsas

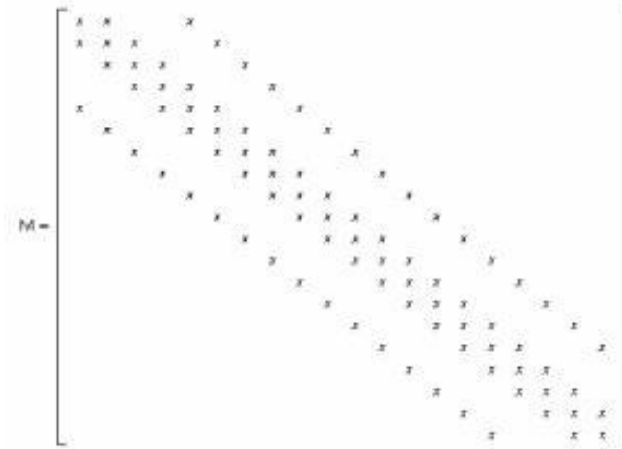
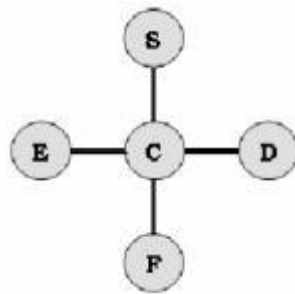
Matrix A : 582 nonzero entries.



$A = LU$: 1950 total nonzero entries.



Formato Diagonal para Matrizes pentadiagonais



$$A = \begin{bmatrix} 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 \end{bmatrix}$$

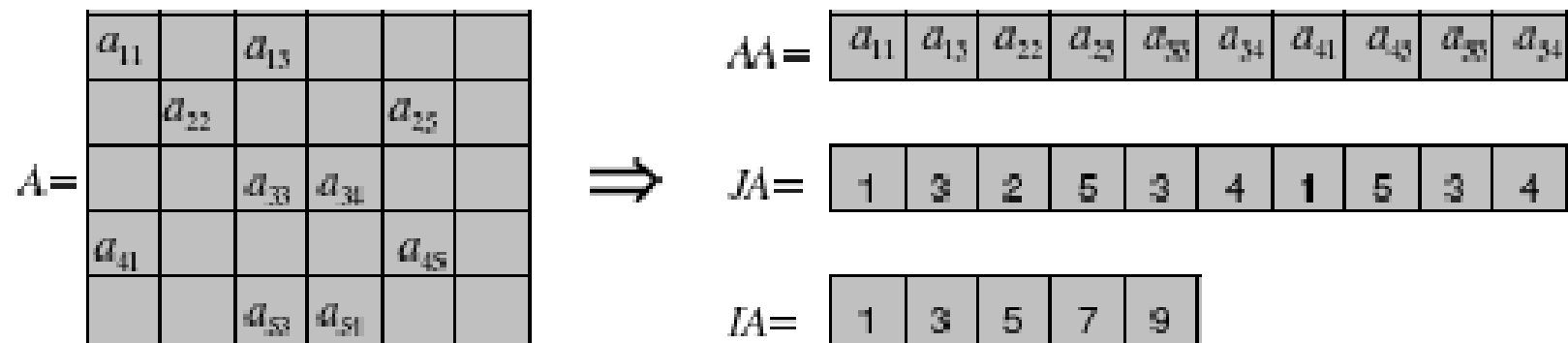
$$\Rightarrow M = \begin{bmatrix} 0 & 0 & 4 & -1 & -1 \\ 0 & -1 & 4 & -1 & -1 \\ 0 & -1 & 4 & -1 & -1 \\ 0 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 & 0 \\ -1 & -1 & 4 & -1 & 0 \\ -1 & -1 & 4 & -1 & 0 \\ -1 & -1 & 4 & 0 & 0 \end{bmatrix}$$

$$\rightarrow Dif = [-4 \ -1 \ 0 \ 1 \ 4]$$

ou

$M(n, 5)$

Formato CSR para Matrizes Esparsas (Compress Sparse Row)



Nnz = número de coeficientes não nulos

Formato CSR (Compress Sparse Row)

$$A = \begin{pmatrix} 10 & 0 & 0 & 0 & -2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 0 & 7 & 8 & 7 & 0 & 0 \\ 3 & 0 & 8 & 7 & 5 & 0 \\ 0 & 8 & 0 & 0 & 0 & 13 \\ 0 & 4 & 0 & 0 & 2 & -1 \end{pmatrix}$$

val	10	-2	3	0	3	7	8	7	3 ... 0	13	4	2	-1
col_ind	1	5	1	2	6	2	3	4	1 ... 5	6	2	5	6

row_ptr	1	3	6	9	13	17	20
---------	---	---	---	---	----	----	----

- n - ordem de A
- nnz - número de coeficientes não nulos
- $2nnz + n + 1$ - número de alocações para armazenar A
- $val(k) = a(i,j)$, $col_ind(k) = j$, $row_ptr(i) \leq k < row_ptr(i+1)$

Formato CDS (Armazena matriz Banda) (Compressed Diagonal Storage)

$$A = \begin{pmatrix} 10 & -3 & 0 & 0 & 0 & 0 \\ 3 & 9 & 6 & 0 & 0 & 0 \\ 0 & 7 & 8 & 7 & 0 & 0 \\ 0 & 0 & 8 & 7 & 5 & 0 \\ 0 & 0 & 0 & 9 & 9 & 13 \\ 0 & 0 & 0 & 0 & 2 & -1 \end{pmatrix}$$

val(:, -1)	0	3	7	8	9	2
val(:, 0)	10	9	8	7	9	-1
val(:, +1)	-3	6	7	5	13	0

$$A = \begin{pmatrix} 10 & -3 & 0 & 1 & 0 & 0 \\ 0 & 9 & 6 & 0 & -2 & 0 \\ 3 & 0 & 8 & 7 & 0 & 4 \\ 0 & 6 & 0 & 7 & 5 & 0 \\ 0 & 0 & 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 5 & 0 & -1 \end{pmatrix}$$

val(:, -2)	0	0	3	6	0	5
val(:, 0)	10	9	8	7	9	-1
val(:, +1)	0	-3	6	7	5	13
val(:, +3)	1	-2	4	0	0	0

- n - ordem de A
- p - número de diagonais não-nulas abaixo da diagonal
- q - número de diagonais não-nulas acima da diagonal

Formato JDS (Armazena matriz Banda) (Jagged Diagonal Storage)

$$\begin{pmatrix} 10 & -3 & 0 & 1 & 0 & 0 \\ 0 & 9 & 6 & 0 & -2 & 0 \\ 3 & 0 & 8 & 7 & 0 & 0 \\ 0 & 6 & 0 & 7 & 5 & 4 \\ 0 & 0 & 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 0 & 5 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -3 & 1 \\ 9 & 6 & -2 \\ 3 & 8 & 7 \\ 6 & 7 & 5 & 4 \\ 9 & 13 \\ 5 & -1 \end{pmatrix}$$

val(:,1)	10	9	3	6	9	5
val(:,2)	-3	6	8	7	13	-1
val(:,3)	1	-2	7	5	0	0
val(:,4)	0	0	0	4	0	0

col_ind(:,1)	1	2	1	2	5	5
col_ind(:,2)	2	3	3	4	6	6
col_ind(:,3)	4	5	4	5	0	0
col_ind(:,4)	0	0	0	6	0	0

- n - ordem de A
- p - número de diagonais não-nulas abaixo da diagonal
- q - número de diagonais não-nulas acima da diagonal

Formato SKL (Armazena Matriz Skyline) (Armazenamento Skyline)

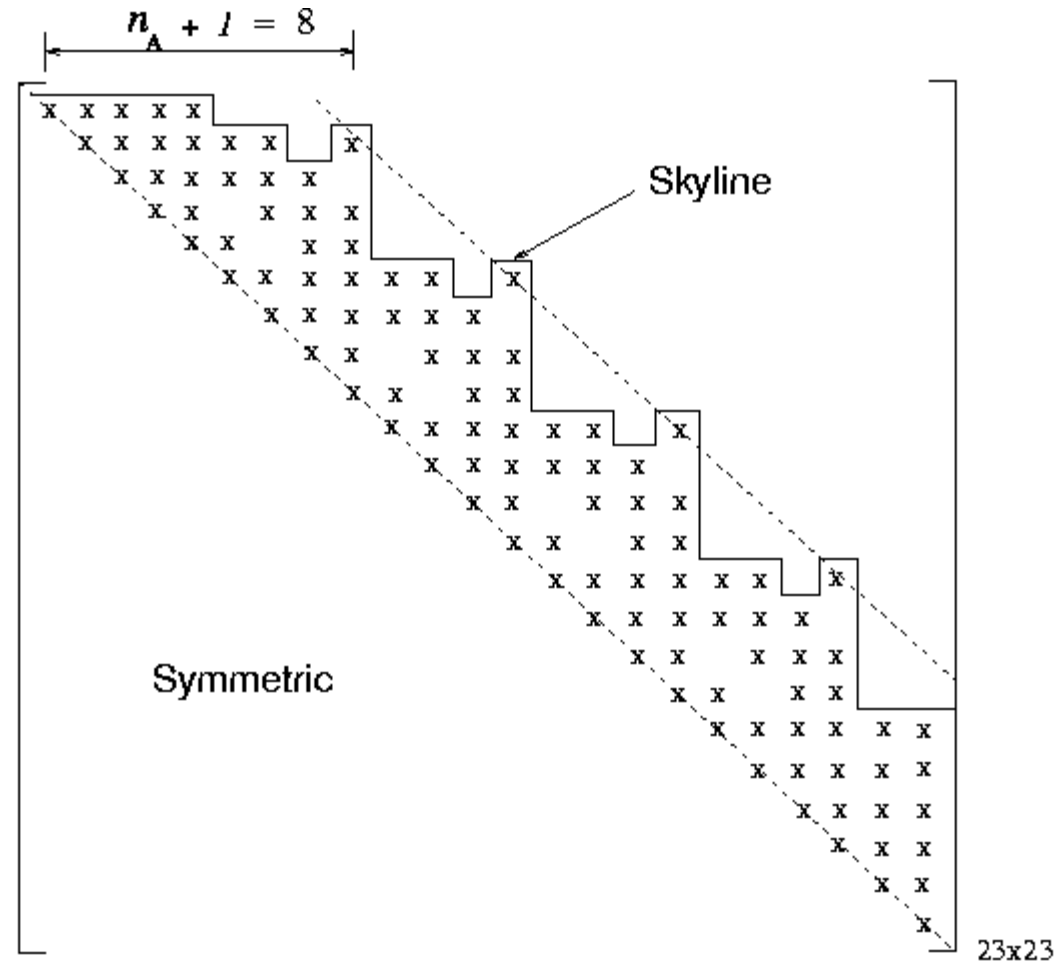


Figure 4.4: The structure of the system matrix A^t of our physics-based fish model, the skyline of A^t and its half-bandwidth n_A . Each "x" represents a non-zero entries.

Formato SKL (Armazena Matriz Skyline) (Armazenamento Skyline)

$$A = \begin{bmatrix} a_{00} & & a_{02} & & \\ & a_{11} & a_{12} & & \\ & & a_{22} & a_{23} & \\ & & & a_{33} & a_{34} \\ & & & & a_{44} \end{bmatrix} \begin{matrix} \Rightarrow n_2 = 0 \\ \Rightarrow n_3 = 2 \end{matrix}$$

Symmetric

$$\bar{A} = \begin{matrix} & \bar{a}_0 & \bar{a}_1 & \bar{a}_2 & \bar{a}_3 & \bar{a}_4 & \bar{a}_5 & \bar{a}_6 & \bar{a}_7 & \bar{a}_8 \\ \begin{bmatrix} a_{00} & a_{11} & a_{22} & a_{12} & a_{02} & a_{33} & a_{23} & a_{44} & a_{34} \end{bmatrix} \end{matrix}$$

$$MAX\bar{A} = \begin{bmatrix} 0 & 1 & 2 & 5 & 7 \end{bmatrix}$$

Figure 4.5: An example of the skyline storage scheme. A is the example system matrix; $\bar{A}$ is the storage array and $MAX\bar{A}$ is the index array.