Towards a Unified Theory of Endurants and Perdurants: UFO-AB

Alessander BOTTI BENEVIDES [a,1], João Paulo A. ALMEIDA [a] and
Giancarlo GUIZZARDI [b]

[a] *NEMO, Computer Science Department, Federal University of Espírito Santo, Brazil*
[b] *Faculty of Computer Science, Free University of Bozen-Bolzano, Italy*

Abstract. The Unified Foundational Ontology (UFO-A) is a foundational ontology about endurants that has been built as a foundation for conceptual modeling, and which focuses on static aspects of endurants. In addition to UFO-A, we have the Unified Foundational Ontology–Part B (UFO-B), a formal theory dealing with the interplay between endurants and the dynamic aspects of reality (e.g., events, processes, causation, dispositions, situations). Given the objective of characterizing this interplay between endurants and perdurants, these two ontologies are meant to form an integral whole. However, currently, they diverge in the way they approach modality. While UFO-A uses a general notion of alethic modality without committing to any notion of time, UFO-B is centered around temporal aspects. As an attempt to address this issue, we here define a translation of the axioms of UFO-A to FOL, and revisit an excerpt of UFO-B in order to accommodate a partial order of time points. With the goal of producing a unified theory, we interpret the alethic modalities of necessity and contingency of UFO-A in terms of this temporal structure, paying a special attention to the interplay between the determinism of a causal nexus and the existence of counterfactual situations. The revisited UFO-B is called UFO-B* and the unified theory is called UFO-AB.

Keywords. alethic modality, endurant, event, foundational ontology, FOL, QS5

1. Introduction

The Unified Foundational Ontology (UFO-A) [8] is an ontology of endurants that was built with the purpose of serving as a foundation for structural conceptual modeling languages. As such, it focuses on static aspects of endurants, including many of their modal aspects. However, this ontology makes no commitment with respect to a particular view of temporal modality. In the Unified Foundational Ontology–Part B (UFO-B) [13,2], in contrast, temporal aspects are fundamental. UFO-B extends UFO-A with a foundational ontology of events, defining a formal theory on the interplay between objects, events, situations, time points, dispositions, processual roles, and the causal nexus. The focus

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[1]Corresponding Author: Alessander Botti Benevides, NEMO, Computer Science Department, Federal University of Espírito Santo, Av. Fernando Ferrari, 514 - Goiabeiras/Vitória, Brazil; E-mail: abbenevides@inf.ufes.br.

of UFO-B is to provide ontological foundations for behavioral conceptual modeling, including for process and project modeling [12,9], situation modeling [1], and discrete event simulation [11].

Regarding their formalizations, UFO-A is expressed in a quantified modal logic (QML) [7] that allows the expression of the alethic modalities of truth (viz., necessity and contingency), and UFO-B is defined in first-order logic (FOL) with the Method of Temporal Arguments (MTA) [26]. In order to enable UFO-A and UFO-B to ground conceptual modeling languages that allow the modeling of both endurants and events, i.e., both static and behavioral aspects of a domain, it is necessary to build a unified formalism for UFO-A and UFO-B. For this task, one move is to interpret the alethic modalities of necessity and contingency in a temporal structure, as done in [3], where the alethic modalities used in UFO-A were interpreted in a branching-time temporal structure.

Differently, [5] interpreted necessity/contingency as immutability/mutability in linear time, which is unable to represent counterfactuals. For instance, while the sentence "John is necessarily a person" can be interpreted in linear time as (*i*) "John is a person" and (*ii*) "John was always a person" and (*iii*) "John will always be a person," (*i*)–(*iii*) do not capture (*iv*) that "John could not cease to be a person," i.e., it is not accidental that John is always a person; being a person is an essential property of John.

Therefore, just interpreting the alethic modalities used in UFO-A into the linear time structure suggested by the current available full formalization of UFO-B (in FOL) will not work; one has to also revisit the axioms of UFO-B, e.g., by representing the dichotomy "actual versus counterfactual" in a partial order of time points (a.k.a. branching-time) as done in [3]. However, one has to deal with the determinism of the causal nexus together with counterfactual situations. In this paper, we define a method for rewriting UFO-A in FOL, with no loss of content, and consistently with a revisited UFO-B.

This paper is structured as follows. Section 2 briefly reviews the notions of alethic and temporal modalities, and their relevance for conceptual modeling and knowledge representation. In Section 3, we show a set of translation rules for transforming the UFO-A QML axioms to FOL. Section 4 adapts UFO-B for a partial order of time points. In Section 5, we present our final considerations.

2. Alethic and Temporal Modalities

Let us first define some notational conventions: (*i*) free variables are implicitly universally quantified; (*ii*) unary predicates are noted in upper camel case `typewriter` type, e.g., "`LivingPerson`;" (*iii*) other predicates and constants are noted in lower camel case `typewriter` type, e.g., "`motherOf`" and "`johnLocke`;" (*iv*) definitions (introduced by "$\triangleq$") "are to be viewed as extraneous conventions of notational abbreviation" [24, p. 72] and are written following the previous cases, but with an *italic* style, e.g., "*Thing*;" and (*v*) the symbols "⌜" and "⌝" denote quasi-quotation.

The alethic modalities of truth distinguish between necessary and contingent truths. Intuitively, a necessary truth is a truth that could not be otherwise, while a contingent truth could be false; or the negation of a necessary truth is contradictory, while the negation of a possible truth is consistent; or a necessary truth is true in all possible worlds, while a possible truth is true in at least one world [14, Ch. 10].

The original specification of UFO-A in [8] uses QML semantically restricted to universal Kripke frames, e.g., in the definition of *rigidity*, *non-rigidity* and *anti-rigidity* [8,

pp. 101–102]. This means that $\ulcorner \Box \varphi \urcorner$ (i.e., $\ulcorner$necessarily $\varphi \urcorner$) is true if and only if (iff) φ is true in all worlds, and $\ulcorner \Diamond \varphi \urcorner$ (i.e., $\ulcorner$possibly $\varphi \urcorner$) is true iff φ is true in at least one world. Quantified **S5** (**QS5**) is a modal logic that is *sound* and *strongly complete* for the class of universal Kripke frames [4, p. 340, Problem 27.2]. This logic was the choice in later presentations of (excerpts of) UFO-A ([23,10]), where there is also an explicit commitment to the Barcan formula and its converse, embracing *possibilistic quantification* [7, § 4.9]. Semantically, possibilistic quantification determines a *fixed* domain of entities for every possible world, which is traditionally associated to the view that the domain includes all the *possibilia*. In this view, it is possible to make a distinction between *existence* and *presence* (at a world). Syntactically, the constants, function applications and variables are *terms*, things that can be predicated upon and that denote entities from the domain of discourse, the set of *existents*. Therefore, semantically, the set of existent things is the domain of quantification of the theory. This view agrees with Quine's dictum that "to be is to be the value of a variable" [25, p. 15]. More specifically, "a theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true" [25, pp. 13–14].

While anything in the domain of quantification of our theory exists, only some entities are `presentAt` a specific world (in other words, are *actual* at a world). Our unary predicate $\ulcorner$`presentAt`$(x)\urcorner$ is a renaming of what Fitting et al. [7] call $\ulcorner \varepsilon(x) \urcorner$, denoting existence in a world. They show how $\ulcorner \varepsilon(x) \urcorner$ can be used to transform formulae from the *varying domain semantics* to *constant domain semantics* [7, p. 106, Definition 4.8.1].[2]

In a sense, in a possibilistic modal theory, a world w "collects" formulae that are true in w. For us, the terms being predicated upon within these formulae must be `presentAt` w. The idea is that $\ulcorner$`presentAt`$(t)\urcorner$ being true at w *allows t* to be mentioned in formulae that are true in w. For this reason, we add to the original axioms of UFO-A the axiom schema (u1), requiring, for each predicate P of arity n in the set $\mathcal{P}$ of predicates of the theory, that whenever a P-predication is true, the arguments of the predication are present.

u1 $\Box \left(P^n(x_i, \ldots, x_j) \to \bigwedge_{k \in \{x_i, \ldots, x_j\}} \texttt{presentAt}(x_k) \right)$, where $P^n \in \mathcal{P}$.

Worlds can be seen as `TimePoints`, the accessibility relation being temporal precedence (e.g., a linear order or a partial order). In this case, necessity can be interpreted as *immutability* and contingency as *possibility within a history*.[3] For instance, the modal logic **KL** is sound for the class of finite strict partial orders. Now, let us assume the causal nexus to be deterministic. Then, a causal chain cannot have bifurcations; each causal chain "happens" over a single history, even if we assume a partial order of time points, like in **KL**. As an example, an analysis of (f1) shows that it requires a `TimePoint` (world) in which `john` is a `Student`, and a `TimePoint` in which `john` is not a `Student`. This is not a problem, since a person can start or cease to be a student in her lifetime (within a single causal chain). Concerning (f2), since `BrazilianBorn` is an immutable (f3) and innate (f4) property, `john` cannot cease to be `BrazilianBorn` within a history. The only way (f2) can be satisfied is by means of two *independent* causal chains,[4] one in

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[2]UFO-A also uses $\ulcorner \varepsilon(x) \urcorner$ in [8, p. 164], but in [23, p. 4], the authors reify times and adopt a temporalized predicate $\ulcorner$`ex`$(x,t)\urcorner$ for existence in worlds at a specific time.

[3]A history/branch is a maximal path, i.e., a maximal *chain* of `TimePoints` within the accessibility relation.

[4]By *independent*, we mean that the causal chains share no `TimePoint`.

which john is always `BrazilianBorn` and other in which he never is.

f1 $\Diamond(\texttt{Student}(x)) \rightarrow \Diamond(\neg\texttt{Student}(x))$

f2 $\Diamond(\texttt{BrazilianBorn}(x)) \rightarrow \Diamond(\neg\texttt{BrazilianBorn}(x))$

f3 $\texttt{BrazilianBorn}(x) \rightarrow \Box(\texttt{BrazilianBorn}(x))$

f4 $\neg\texttt{BrazilianBorn}(x) \rightarrow \Box(\neg\texttt{BrazilianBorn}(x))$

Types (universals) are reified in both UFO-A and UFO-B. In UFO-A, the predicate "::" represents instantiation (see [8, p. 162], [23, p. 3]), while in UFO-B it is represented by "`instantiates`" [2]. That said, properties like `Student` and `BrazilianBorn` are called *anti-rigid* and can be captured in **QS5** by (u2), where rigidity is defined in **QS5** by (u3). In **QKL**, (u2) and (u3) define immutability and mutability, respectively.

u2 $AntiRigid(p) \triangleq x::p \rightarrow \Diamond(\neg(x::p))$

u3 $Rigid(p) \triangleq x::p \rightarrow \Box(\texttt{presentAt}(x) \rightarrow x::p)$

3. Capturing the Alethic Modality of UFO-A in FOL

While in Section 2 we discussed modal notions, our aim is to build a FOL theory. Here, we show a method for translating a **QS5** theory (like UFO-A) to FOL. Let $\mathcal{T}$ be a **QS5** theory with a countably infinite set of variables $\{x_0, \ldots, x_{\lambda-1}, x_\lambda, \ldots\}$. In practice, the theory $\mathcal{T}$ only uses variables in $\{x_0, \ldots, x_{\lambda-1}\}$, where $\lambda-1$ is the biggest index (a natural number) of variables used in axioms. This means that the variable x_λ is never used in $\mathcal{T}$. By reifying worlds in the domain of quantification by means of the variable x_λ, it is possible to rewrite the axioms of $\mathcal{T}$ into a FOL theory $\overline{\mathcal{T}}$. We define a set of translation rules in $\mathcal{A} = \{(r1), \ldots, (r5)\}$. Predicates can be rewritten with an additional argument, as in (r1). The translation rules (r2)–(r5) define the recursive steps. **S5** has the nice property that every well-formed formula (wff) is equivalent to a wff with *modal depth* ≤ 1, i.e., a wff in which no modal operator falls in the scope of another modal operator (see [4, p. 340, Problem 27.3]). There is an algorithm (that always halts) for finding such a wff, and we denote its application to a wff φ as $\mathring{\varphi}$. This allows us to use only one variable, x_λ, to quantify over worlds in the translation rule (r5).

r1 $\overline{P^n(x_i, \ldots, x_j)} \triangleq P^{n+1}(x_i, \ldots, x_j, x_\lambda)$, where $P^n \in \mathcal{P}$ and n is the arity of P.

r2 $\overline{\neg\varphi} \triangleq \neg\overline{\varphi}$

r3 $\overline{\varphi \wedge \alpha} \triangleq \overline{\varphi} \wedge \overline{\alpha}$

r4 $\overline{\forall x_i(\varphi)} \triangleq \forall x_i(\overline{\varphi})$

r5 $\overline{\Box\varphi} \triangleq \forall x_\lambda(\overline{\mathring{\varphi}})$

4. A Revisited Excerpt of UFO-B

Here, we revisit an excerpt of UFO-B (a FOL theory) in order to accommodate a partial order of time points. Except for the Allen relations, which require a branching-time counterpart (see [15]), the rest of UFO-B is unaffected by the work presented here.

4.1. Time

While UFO-B [13,2] suggests a discrete linear ordering of `TimePoints`, here we adopt a discrete strict partial order "precedes" over `TimePoints` (t1). Therefore, `precedes` is irreflexive (t2), transitive (t3) and asymmetric (a theorem). A `TimePoint` can be an *immediateSucessorOf* another `TimePoint` (t4). (t5) defines when two `TimePoints` can be seen as an *interval*, a part of a history.[5] (t6) defines when two `TimePoints` are *comparable*. (t7) defines when one interval is a *subIntervalOf* of another interval. (t8) provides a predicate `sameLength` for comparing the length of two intervals.

t1 $\text{precedes}(x,y) \rightarrow \text{TimePoint}(x) \wedge \text{TimePoint}(y)$

t2 $\neg \text{precedes}(x,x)$

t3 $\text{precedes}(x,y) \wedge \text{precedes}(y,z) \rightarrow \text{precedes}(x,z)$

t4 $immediateSucessorOf(x,y) \triangleq \text{precedes}(y,x) \wedge \neg \exists z (\text{precedes}(z,x) \wedge$
$\text{precedes}(y,z))$

t5 $interval(x,y) \triangleq x = y \vee \text{precedes}(x,y)$

t6 $comparable(x,y) \triangleq interval(x,y) \vee \text{precedes}(y,x)$

t7 $subIntervalOf(x_b,x_e,y_b,y_e) \triangleq interval(x_b,x_e) \wedge interval(y_b,y_e) \wedge$
$((x_b = y_b \vee \text{precedes}(y_b,x_b)) \wedge$
$(x_e = y_e \vee \text{precedes}(x_e,y_e))$

t8 $\text{sameLength}(x_b,x_e,y_b,y_e) \leftrightarrow interval(x_b,x_e) \wedge interval(y_b,y_e) \wedge$
$((x_b = x_e \wedge y_b = y_e) \vee$
$\exists z,z'(immediateSucessorOf(z,x_b) \wedge$
$immediateSucessorOf(z',y_b) \wedge$
$\text{sameLength}(z,x_e,z',y_e)))$

Moreover, a branching after a `TimePoint` t must be justified: there must be a difference between the set of things that are `presentAt` t' and the ones `presentAt` t''. More specifically, there must be (i) an event with `beginPoint` or `endPoint` in t' but not in t'', or (ii) a situation that `obtainsIn` t' but not in t'' (t9). Notice that (i) includes the case of an object being created or terminated (by an event with `endPoint`) in t' but not in t''. Finally, only entities other than `TimePoints` can be `presentAt` `TimePoints` (t10), and, for ontological parsimony, there is at least one entity present at a `TimePoint` (t11).

t9 $t' \neq t'' \wedge immediateSucessorOf(t',t) \wedge immediateSucessorOf(t'',t) \rightarrow$
$\exists x((\text{beginPoint}(x,t') \wedge \neg \text{beginPoint}(x,t'')) \vee$
$(\text{endPoint}(x,t') \wedge \neg \text{endPoint}(x,t'')) \vee$
$(\text{obtainsIn}(x,t') \wedge \neg \text{obtainsIn}(x,t'')))$

t10 $\text{presentAt}(x,y) \rightarrow \neg \text{TimePoint}(x) \wedge \text{TimePoint}(y)$

t11 $\text{TimePoint}(x) \rightarrow \exists y(\text{presentAt}(y,x))$

4.2. Endurants

`Endurants` (e.g., a cat, the Moon, Mick Jagger) *are in time*, in the sense that it is possible to refer to `john` as a child and `john` as an adult as the *same* entity, which under-

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[5]Although we informally talk about *histories*, we do not reify them in our theory. We leave as future work investigating whether reifying histories would be of benefit.

goes changes over time. For ontological parsimony, each `Endurant` is `presentAt` some `TimePoint` (e1).

e1 $\mathtt{Endurant}(x) \rightarrow \exists y(\mathtt{presentAt}(x,y))$

4.3. Events

Fallings cannot fall, deaths cannot die, changes cannot change. Although we do not commit to the notion of `Events` as changes (which is the main assumption of Lombard in [22]), we agree that non-relational changes *are* events. `Events` cannot undergo non-relational changes.[6] `Events` *happen in time*. Since Lombard's theory and ours share the view that `Events` have inter-world identity, i.e., an `Event` can occur over different histories, it is fruitful to compare his theory with ours.

While for Lombard [20, p. 354],[17, p. 454] `Events` could have been different from what they were by having contingent temporal parts, i.e., a single `Event` could "aggregate" different *occurrences*,[7] each of which sharing the same essential parts, but differing in some contingent parts; in our theory, `Events` have no alethic modal properties. An `Event` could not have been different from what it was. Nonetheless, it makes sense to allow `Events` to have temporal branchings, occurring over incomparable `TimePoints`, but only for the special cases where the reason for the temporal branching is external to the `Event`, i.e., the `Event` happens the same way over the different histories. Therefore, `Events` may have not exactly one, but more than one `beginPoints` and `endPoints` (e2), (e3). Moreover, `Events` are non-instantaneous (e3), which is compatible with Lombard in [20, p. 348]. `Events` can only happen over different histories because it would not make sense to postulate the existence of two events in cases where it is impossible to distinguish the two occurrences based solely on their non-relational aspects.[8]

e2 $\mathtt{beginPoint}(e,t) \lor \mathtt{endPoint}(e,t) \rightarrow (\mathtt{Event}(e) \land \mathtt{TimePoint}(t))$
e3 $\mathtt{Event}(e) \rightarrow \exists t, t'(\mathtt{beginPoint}(e,t) \land \mathtt{endPoint}(e,t'))$

For Lombard in [21, p.283], `Events` cannot recur. We have a stronger position in that `Events` are nonintermittent, and that is why they cannot recur over a history, having at most one `beginPoint` in a history (e4). Moreover, `Events` must end, i.e., in each history in which the `Event` begins, it must end (e5). Similarly, `Events` must begin (e6). Finally, whenever an `Event` occurs over two histories, the two occurrences must have the same length (e7), which also agrees with Lombard's view [18, p. 9, Footnote 11].

e4 $t \neq t' \land comparable(t,t') \land \mathtt{beginPoint}(e,t) \rightarrow \neg\mathtt{beginPoint}(e,t')$
e5 $\mathtt{beginPoint}(e,t) \land comparable(t,t') \rightarrow \exists t''(comparable(t',t'') \land$
$\mathtt{endPoint}(e,t''))$
e6 $\mathtt{endPoint}(e,t) \land comparable(t,t') \rightarrow \exists t''(comparable(t',t'') \land$
$\mathtt{beginPoint}(e,t''))$
e7 $\mathtt{beginPoint}(e,x_b) \land \mathtt{endPoint}(e,x_e) \land interval(x_b,x_e) \land \mathtt{beginPoint}(e,y_b) \land$
$\mathtt{endPoint}(e,y_e) \land interval(y_b,y_e) \rightarrow \mathtt{sameLength}(x_b,x_e,y_b,y_e)$

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[6]See [16] for (non-)relational changes.
[7]Although we informally talk about *occurrences* of `Events`, we do not reify them in our theory.
[8]Again, "non-relational" as in [16]. Moreover, we think that identity criteria should be based solely on intrinsic properties; e.g., we disagree with the causality-based identity criterion for `Events` proposed in [6].

A `Situation` is the truthmaker of a synchronic proposition. For instance, the proposition "John is awake now" is made true by a `Situation` such as `thatJohnIsAwakeNow`. This view is derived from *logical atomism*, but `Situations` differ from Russell's facts and Wittgenstein's states of affairs by being synchronic. In our view, reality-at-time-*t* is partitioned into atomic `Situations`, and starting from them, one can describe reality-at-time-*t* in a bottom-up manner by means of atomic synchronic propositions.

Since the atomic synchronic propositions describing atomic `Situations` are the most fine-grained pieces of synchronic information one can get, such propositions should be logically independent. Since we are not reifying propositions, we pose this constraint on `Situations`: Given two concomitant atomic `Situations`, knowing the structure of one is insufficient for deriving the structure of the other (f5).

f5 $s \neq s' \wedge \mathtt{concomitant}(s,s') \wedge \mathtt{instanceOf}(s,S) \nvdash \mathtt{instanceOf}(s',S')$

However, capturing syntactic consequence ("$\vdash$" and "$\nvdash$") requires a reflection mechanism, like Gödel functions, where wffs are reified as terms of the theory. Such a requirement would deviate our attention into the technicalities of a theory of first-order arithmetic. Therefore, we will not formally capture the nature of `Situations`, but only their interplay with `Endurants` and `Events`. Since we are capturing the alethic modality by means of a partial order of time points, *possible* `Situations` must obtain (s1) in at least one `TimePoint` in a history that entertains such a possibility (s2). Moreover, `Situations` should not re-obtain over a history (s3), they are *bound to* specific `TimePoints`.[9]

s1 $\mathtt{obtainsIn}(x,y) \rightarrow (\mathtt{Situation}(x) \wedge \mathtt{TimePoint}(y))$
s2 $\mathtt{Situation}(x) \rightarrow \exists y(\mathtt{obtainsIn}(x,y))$
s3 $t \neq t' \wedge \mathit{comparable}(t,t') \wedge \mathtt{obtainsIn}(s,t) \rightarrow \neg\mathtt{obtainsIn}(s,t')$

`Situations` are similar to *enablers* in [19, §V], an obtaining `Situation` can be a necessary but not a sufficient condition for an `Event` to occur. `Situations` have no causal power, only `Events` do. An `Event` `bringsAbout` exactly one `Situation` (s4), (s5), which holds in all `endPoints` of the `Event` (s6). Also, an `Event` is triggered by exactly one `Situation` (s7),(s8), which holds in all `beginPoints` of the `Event` (s9). Usually, a `Situation` is considered a *Fact* (w.r.t. a history) iff the situation `obtainsIn` a `TimePoint` (of the history). Since we are not reifying histories, we define a `Situation` as a *Fact* w.r.t. a `TimePoint` t iff there is a t' *comparable* with t—so that t and t' are in a path of a history—and such that the `Situation` `obtainsIn` t' (s10). Finally, except for `TimePoints` with no preceding `TimePoint`, if a `Situation` s `obtainsIn` t, s must be brought about by an `Event` with `endPoint` at t (s11).

s4 $\mathtt{bringsAbout}(e,s) \rightarrow (\mathtt{Event}(e) \wedge \mathtt{Situation}(s))$
s5 $\mathtt{Event}(e) \rightarrow \exists!s(\mathtt{bringsAbout}(e,s))$
s6 $\mathtt{bringsAbout}(e,s) \wedge \mathtt{endPoint}(e,t) \rightarrow \mathtt{obtainsIn}(s,t)$
s7 $\mathtt{triggers}(s,e) \rightarrow (\mathtt{Situation}(s) \wedge \mathtt{Event}(e))$
s8 $\mathtt{Event}(e) \rightarrow \exists!s(\mathtt{triggers}(s,e))$
s9 $\mathtt{triggers}(s,e) \wedge \mathtt{obtainsIn}(s,t) \rightarrow \mathtt{beginPoint}(e,t)$

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[9]We have no strong objection against removing (s3), leaving such an analysis for future work.

s10 $Fact(s,t) \triangleq \exists t'(comparable(t,t') \wedge \mathtt{obtainsIn}(s,t'))$

s11 $\exists t'(\mathtt{precedes}(t',t)) \wedge \mathtt{obtainsIn}(s,t) \rightarrow$
$$\exists e(\mathtt{bringsAbout}(e,s) \wedge \mathtt{endPoint}(e,t))$$

4.5. Time is a Set of Lines

The so called *essentiality thesis* states that an `Event` must occur at the same `TimePoints` in all histories in which it occurs [18]. In [18, pp. 7–13], Lombard shows an argument against the *in*essentiality thesis, where he presupposes that determinism must be false for the inessentiality to make sense. Here, we show that one must not give up determinism in order to commit to the inessentiality thesis. As an example for the inessentiality thesis, let an event e start in a `TimePoint` t_i in a history h that begins at t_0, and e also starts in a `TimePoint` t_i' in a history h' that begins at t_0' and s.t. the length of the interval $[t_0,t_i]$ is different from the length of $[t_0',t_i']$. Determinism can still hold when h and h' differ in the `Situations` that `obtainsIn` t_0 and t_0'. Moreover, determinism also holds with the inessentiality thesis when two histories have no beginning. Non-surprisingly, the notion of contingency that is lost due to the view of time as a line in [5] is "regained" by allowing the existence of independent time lines, which are able to entertain counterfactual truths.

This means that `precedes` is not a strict total order because, while it is asymmetric and transitive, it is not *semiconnexOver* `TimePoints` (f6). We prove that `precedes` has a weaker property: *transitivity of comparability* (f7).

f6 $semiconnexOver[\mathrm{R},\mathrm{S}] \triangleq (\mathrm{S}(x) \wedge \mathrm{S}(y) \wedge x \neq y) \rightarrow (\mathrm{R}(x,y) \vee \mathrm{R}(y,x))$

f7 $(comparable(x,y) \wedge comparable(y,z)) \rightarrow comparable(x,z)$

More specifically, we prove that $\{(t1), \ldots, (t11), (e1), \ldots, (e7), (s1), \ldots, (s11)\}$ entails a temporal structure that is a set of lines. The proof goes as follows. Assume that there is a bifurcation at `TimePoint` t, which is immediately succeeded by both t' and t''. From (t9), there must be (i) an event with `beginPoint` or `endPoint` in t' but not in t'', or (ii) a situation that `obtainsIn` t' but not in t''.

In case (i), if there is an event e with `beginPoint` in t' but not in t'', from $\{(s8),$ $(s9)\}$, there is a situation that `obtainsIn` t' and `triggers` e, but that does not obtainsIn t'' (otherwise, by (s9), e would also have a `beginPoint` at t''). Then, (a) from (s11), since the situation `obtainsIn` t', it must be brought about by an event e' with `endPoint` in t' but not in t'' (otherwise, by (s6), the situation would also obtain in t''). Then, (b) from (e6), e' must have a `beginPoint` in t or earlier. Since e' started before or in t, it must end in both branches, i.e., not only in t' but also in all histories containing t'' (e5). Moreover, from (e7), all the occurrences of e' must have the same length. Therefore, e' must have an `endPoint` at t'', a contradiction. On the other possibility, if there is an event with `endPoint` in t' but not in t'', then the proof goes as in (b).

In case (ii), the proof goes as in (a).

4.6. Causality

The `Event john'sFall bringsAbout` the `Situation thatJohnIsHurt`, which `triggers` the `Event john'sCry`. So, `john'sFall` *directlyCauses* `john'sCry`, as defined in (c1). The transitive closure of `directlyCauses` is a useful generalization, encompassing indirect causes. Since transitive closure is inexpressible in FOL, we have `causes` as a predicate (see (c2)), constraining it to be asymmetric (c3).

c1 $directlyCauses(e,e') \triangleq \exists s(\text{bringsAbout}(e,s) \wedge \text{triggers}(s,e'))$

c2 $\text{causes}(e,e'') \leftrightarrow (directlyCauses(e,e'') \vee \exists e'(\text{causes}(e,e') \wedge \text{causes}(e',e'')))$

c3 $\text{causes}(e,e') \rightarrow \neg\text{causes}(e',e)$

4.7. Event Mereology

We will not repeat here the axioms of the extensional atomic event mereology of UFO-B (see [13,2]). To that axiomatization, we add the following axioms. A part of an event must occur in all histories in which the whole occurs (m1). The ordering of event parts must be the same in all histories (m2) (literally, (m2) states that the length of the interval from a beginning of the whole to a comparable beginning of the part is constant).

m1 $\text{hasPart}(e,e') \wedge \text{beginPoint}(e,x_b) \wedge \text{endPoint}(e,x_e) \wedge interval(x_b,x_e) \rightarrow$
$\exists y_b, y_e(\text{beginPoint}(e',y_b) \wedge \text{endPoint}(e',y_e) \wedge subIntervalOf(y_b,y_e,x_b,x_e))$

m2 $\text{hasPart}(e,e') \wedge \text{beginPoint}(e,t) \wedge \text{beginPoint}(e',t') \wedge comparable(t,t') \wedge$
$\text{beginPoint}(e',t'') \wedge comparable(t,t'') \rightarrow \text{sameLength}(t,t',t,t'')$

4.8. UFO-B*

UFO-B*, defined by (a1), is the version of UFO-B (from [2]) where `timePoints` are partially ordered. UFO-AB, the unification of UFO-A and UFO-B, is defined by (a2).

a1 UFO-B* = (UFO-B $\setminus \{$(T1), ..., (T14'), (S1'), ..., (S8)$\}) \cup \{$(t1), ..., (t11), (e1), ..., (e7), (s1), ..., (s11), (c1), ..., (c3), (m1), (m2)$\}$

a2 UFO-AB = $\overline{\text{UFO-A}} \cup \{$(u1)$\} \cup$ UFO-B*

5. Conclusion

The Unified Foundational Ontology (UFO) was built with the goal of providing foundations for Conceptual Modeling. In order to do that, it must be able to address both structural and dynamic aspects of reality, i.e., it must be able to characterize ontological aspects of *endurants*, *perdurants*, as well as their interplay. These two realms of reality have so far been addressed by two formally independent theories, namely, UFO-A and UFO-B, respectively. In order to produce a theory unifying these two, one must be able to address the different treatment of modality in these two approaches, i.e., one must be able to reconcile the alethic modalities of necessity and contingency in UFO-A with models of time that are essential for talking about events, processes and causation in UFO-B.

This paper presents a contribution in this direction. Firstly, we define a (*i*) translation of the axioms of UFO-A (in [8]) to FOL. We then (*ii*) revisit an excerpt of UFO-B (in [2]) to accommodate a partial order of time points, thus, producing a theory called UFO-B*. Finally, leveraging on (*i*) and (*ii*), we produce a formal unified theory called UFO-AB.

Acknowledgments

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