Relations in Ontology-Driven Conceptual Modeling

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Abstract. For over a decade now, a community of researchers has contributed to the ontological foundations of Conceptual Modeling by participating to the development of the Unified Foundational Ontology (UFO) and the UFO-based modeling language OntoUML, which have been successfully employed in a number of different sectors. The empirical feedback from these experiences led us to reconsider UFO's *theory of relations*, proposing a new theory that has already been applied to model subtle notions in the business domain, such as value, risk, service, and contract. In this paper, we advance a first formal characterization of this new theory, which is then used to design a new metamodel for OntoUML.

Keywords: Relations · Relationships · Ontology-Driven Conceptual Modeling · OntoUML · UFO

1 Introduction

Applied philosophical theories have gained an increasing importance in conceptual modeling in the past decades, supporting different modeling approaches. More specifically, the notion of foundational ontologies emerged in the form of comprehensive theories seeking to consistently define fundamental concepts in the field, e.g., *types and taxonomic structures*, *roles and relational properties*, *part-whole relations*, *multi-level structures*, etc. An ontology developed with the goal of providing foundations for all these major conceptual modeling constructs is UFO (Unified Foundational Ontology) [10, 14]. Over the years, UFO has been employed for the evaluation and (re)design of conceptual modeling languages and reference models in a variety of domains [12]. One of the main applications of UFO has been the design of a general-purpose language for *ontology-driven conceptual modeling* (ODCM) OntoUML. Following a systematic language engineering process [10], OntoUML has been created as a revised version of UML such that: (i) its modeling primitives reflect the ontological distinctions put forth by UFO; (ii) its metamodel includes semantically-motivated syntactic constraints that reflect the axiomatization of UFO. Research shows that UFO and OntoUML are among the most used foundational ontology and modeling language in the ODCM literature, respectively [24]. Moreover, empirical evidence shows that OntoUML significantly contributes to improving the quality of conceptual models without requiring an additional

effort to produce them. For instance, Verdonck's work [23] reports on a modeling experiment conducted with 100 participants in two countries showing the advantages of OntoUML when compared to a classical conceptual modeling language (EER).

The observations of the way OntoUML was applied over the years, conducted by several groups in a variety of domains, are a fruitful empirical source of knowledge on the language and its foundations [12]. In particular, we observed a number of different ways in which people did slightly subvert the syntax of the language, ultimately creating what we called *systematic subversions* [10]. These "subversions" did (purposefully) produce models that were grammatically incorrect, but which were needed to express intended meanings that could not be expressed otherwise. We labeled them as "systematic" because they were recurring in the works of different authors that, independently of each other, were subverting the language in the same manner and with the same modeling intention. One of these "subversions" led us to reconsider UFO's *theory of relations* [6,7], proposing a new theory that has been applied to model a number of different notions, including *value*, *risk*, *preference*, *service* and *contract* [5,17,21], whose (preliminary) formal characterization will be presented here for the first time.

Relations are fundamental for conceptual modeling, and, for many years, researchers have been looking at ontological theories to account for relevant distinctions among them, and provide ways to *talk* of them by means of *reification* mechanisms [1,3,11,25]. In this paper, leveraging on previous revisitations of UFO's notion of *relator* [6,7,9], we present UFO's new theory of relations as well as its OntoUML counterpart (a suitable fragment of OntoUML 2.0 [13]). As we demonstrate, this new theory is much richer than the existing proposals in the literature, with important consequences for conceptual modeling practice.

The contributions of this paper are three-fold. First, we present a first formal characterization for this new theory of relations. Second, following the same ontology-based language engineering approach that was used to create the original version of OntoUML [10], we employ this new formalized version of the theory to propose an enhanced metamodel for OntoUML 2.0. Finally, we employ this metamodel to implement a model construction and verification tool for OntoUML 2.0. The remainder of this paper is organized as follows: Section 2 provides the background for the paper briefly reviewing OntoUML and UFO, including its new ontological theory of relations. The section also briefly analyzes the limitations of the original version of OntoUML and its underlying theory with respect to the conceptual modeling of relations; Section 3 presents a rich formalization of the new theory, accounting for relators and for different kinds of relations; Section 4 presents the OntoUML 2.0 relations metamodel and the modeling patterns [9] for the various kinds of relations, incorporated into the language; Section 5 briefly discusses related work and presents our final considerations.

2 Background: UFO, OntoUML and a New Theory of Relations

OntoUML was originally designed to represent invariant structures of endurants (object-like entities) and their relations, reflecting the ontological distinctions in UFO. In this foundational ontology, endurants are partitioned into *substantials* and *moments*. Substantials are existentially independent individuals, e.g., a car, a person, or an organi-

zation. In contrast, moments are specific *aspects* of individuals that are existentially dependent on them, such as (a) a flower's color or (b) Bob's headache, and may be also existentially dependent on other individuals, as in the case of (c) John's love for Mary or (d) the marriage between John and Mary. The specific sort of existential dependence connecting moments to their *bearers* is termed *inherence*. Each of these examples of moments reflects a different category within UFO (Fig. 2)[4]: (a) is an example of a *quality*, a particular aspect of an individual that may be useful to compare it with other individuals, on the basis of the value it takes in a certain quality space (for instance, a position within the RGB spectrum) [10]; (b) and (c) are examples of *modes*, i.e., aspects that can have their own qualities; in particular, (b) is an *intrinsic mode*, since it only depends on its bearer, while (c) is an example of *extrinsic mode*, also called *externally dependent mode* since, besides inhering in John, it is also existentially dependent on Mary, accounting for a one-sided relationship between John and Mary; finally, (d) may be seen as a sum of externally dependent modes accounting for reciprocal one-sided relationships (such as John's love for Mary, John's obligations towards Mary, and the reciprocal relationships on Mary's side), which form altogether a complex two-sided relationship. Qualities and intrinsic modes are collectively called *intrinsic moments*, as they are intrinsic to their bearers. *Extrinsic modes* include externally dependent modes and mereological sums of two or more externally dependent modes, which are collectively called *relators*.

In OntoUML, an association stereotyped as *«characterization»* represents (at the type-level) the existence of an *inherence* relation connecting the instances of those types, i.e., connecting intrinsic moments and their bearers. Analogously, associations stereotyped as *«mediation»* are used to connect relators to their relata.[5] Both *«characterization»* and *«mediation»* are special cases of *existential dependence* [10].

The original version of UFO made a fundamental distinction between *formal* and *material* relations. Intuitively, the former were assumed to hold "directly without any further intervening individual", while the latter required the existence of an intervening individual. Formally, material relations where defined as presupposing the existence of a *relator* composed of externally dependent modes (each inhering in one relatum and externally dependent on the other) all historically dependent on a common external *foundation event*. Formal relations where defined as relations that are not material. Typical examples of material relations where *married-with* or *employed-by*, while formal relations included inherence, mediation and parthood, as well as *comparative relations* such as *heavier-than*.

Figure 1 illustrates how these relations appear in OntoUML in its current version. Stereotypes are used to distinguish «formal» and «material» relations, as well as to identify «characterization» and «mediation». Moreover, the *derivation* relation is represented by a dashed line connecting the relator type Marriage and the relation married-with, such that we have that the tuple $\langle John, Mary \rangle$ instantiates the latter iff it is mediated by an instance of the former, i.e., by a particular instance of Marriage.

[4] The taxonomy we are describing, depicted in Fig. 2, has been slightly changed with respect to to UFO's original one.

[5] We stick to the term 'mediation' just for reasons of compatibility with previous papers. In the past we also used 'involvement', which is perhaps a better terminological alternative.

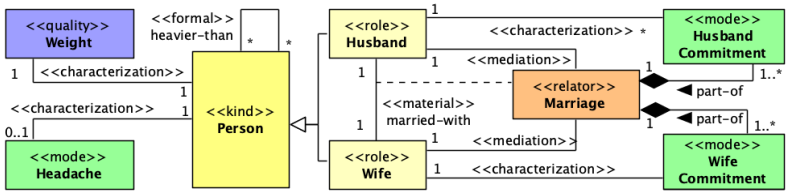

Fig. 1. Example of relations in the current version of OntoUML.

2.1 Limitations of UFO/OntoUML 1.0 regarding the modeling of relations

In the original version of UFO, the distinction between formal and material was exhaustive, i.e., all relations that were not consider material (i.e., mediated by an external entity) were automatically classified as formal. Take, for example, the ternary relation of *Economic Preference* [18], defined between an agent and two resources. This relation is completely grounded on two modes of the agent, namely, two *value ascriptions* made by that agent with respect to those resources. However, in this case, there is no property that is acquired by these resources in virtue of being preferred (or deprecated) by that agent! The only entity that has relational properties grounding that relation is that agent. Now, since in UFO relators are aggregations of externally dependent modes of all relata, the sum of the valuations of this agent is not a relator and, hence, *preference* cannot be considered a *material* relation. As a consequence, it must be considered a formal relation and, hence, classified together with relations as diverse as *being-older* than and *existential dependence*.

So, the original UFO theory of relations was too restrictive (w.r.t. *material relations*), proscribing the existence of *single-side relational moments*. On the other hand, the theory was too permissive (w.r.t. *formal relations*), including in the same class, for example, relations holding directly as soon as their relata exist (e.g., existential dependence, inherence, instantiation) and relations reducible to intrinsic properties of the relata (e.g., *comparative relations* such as *older-than*), as well as the so-called *mere Cambridge relations* [7], e.g., economic preference or value ascription.

A practical drawback of the aforementioned *restrictive* aspect is the difficulty in modeling relations based on single-side relational qualities, which abound in practice. For example, this shortcoming of the language has caused several experienced researchers to radically diverge regarding the modeling of standard relations in the ISO REA framework [8]. Moreover, since *relationship reification* was restricted to material relations, the modeling benefits of reifying other types of relationships would often escape modeler's attentions. Indeed, in a previous paper [12] some of us discussed the benefits of reifying comparative relations such as *heavier-than*, for example, to track the changes in the weight variation of two physical objects in time.

A practical drawback of the *permissive* aspects of the original theory is that, since relations of different sorts were grouped in the same class, the constraints in the language for the modeling of these relations were basically non-existing, namely, the use of standard associations with a stereotype «formal». As a consequence, for example, when modeling *comparative relations*, there was nothing in the language forcing the

modeler to pay attention to the existence of particular qualities in the relata that would ground that relation (e.g., in the way that *heavier-than* should be grounded in the individual weights of the relata). Futhermore, as demonstrated by [20], after analyzing a repository of dozens of OntoUML models, a frequent anti-pattern in ontology-driven conceptual modeling is the use of the «formal» stereotype to model relations neglecting a deeper analysis of their nature, exactly because of the lack of additional constraints associated with that stereotype.

As a final limitation, we highlight that although the original OntoUML metamodel explicitly represents different forms of existential dependence (e.g., inherence, mediation), it does not provide any native support for other forms of *specific dependence*, which recurrently appeared in practical domains. The most prominent of these being *external dependence* (for example, in the domains of Service [5] and Risk [21]) and *historical dependence* (for an example, in many *ontologies of artifacts* [26]).

2.2 Extending UFO's Original Theory of Relations

In a recent series of papers [6, 7, 9], Guarino and Guizzardi revisited the ontological nature of relations and relationships by focusing on the following question: if a relation *R* holds for relata *x* and *y*, what is there in the world that is the *truthmaker* of this relational sentence, i.e., what is responsible for its truth? What is the nature of such truthmaker? By relying on distinctions with respect to different types of truthmakers, the authors proposed a *typology of relation types* that goes beyond the original distinction between *formal* and *material*, relying on two orthogonal distinctions: *internal*/*external* and *descriptive*/*non-descriptive*.

So far, we kept refining our own understanding of these two distinctions in an informal way, resulting in changes in the way some relations where classified. Indeed, as discussed in [9], the philosophical terms used for such distinctions are often used in different ways, so that it is difficult to draw an accurate picture. This is the reason why, in this paper, we decided to aim at a rigorous axiomatic characterization, both to clarify the ontological assumptions behind these distinctions and to allow us to formally derive the constraints to be implemented in the new version of OntoUML (OntoUML 2.0) in order to enforce ontologically well-founded modeling patterns (Table 1).

According to a definition originally due to Russell [19], **internal relations** are relations definable in terms of the intrinsic properties of their relata. A classic example are comparative relations. They may hold either in virtue of intrinsic moments of the relata (e.g., John is taller than Mary because of their intrinsic height qualities) or just in virtue of the way the relata intrinsically are, without involving their qualities (e.g., John's height is greater than Mary's height). **External relations**, in contrast, cannot be just defined in terms of intrinsic properties of their relata. This means that they either: (i) rely on at least one property of a relatum that depends on something else (typically, the other relatum). The classic example is the marriage relation, whose truthmaker is composed of the mutual commitments and obligations of the partners, which are modes inhering in each of them and externally dependent on the other one; (ii) are primitive non-analyzable relations (e.g., existential dependence and its specializations such as *inherence* and *mediation*). In summary, in the case of an external relation connecting *x* and

y, there is something about *x* that requires the existence of *y*. This externally dependent entity is either a moment of *x* or *x* itself.

In an orthogonal dimension, **descriptive relations** hold in virtue of some *moment* (aspect) of the relata. For example, both *in love with* and *heavier than* between people hold because of specific moments of their relata (a love mode in the first case; weight qualities in the second case). In contrast, **non-descriptive relations** hold because of the entity as a whole (e.g., *greater than* between two qualities, such as weight or height). Each combination of the two distinctions (i.e., *internal*/*external* and *descriptive*/*non-descriptive*) corresponds to an interesting class of relations. For example, in this account of descriptiveness, a *historical dependence* relation such as *born in* turns out to be non-descriptive (since it does not involve an intrinsic quality of its relata) and external, since what makes it true is something external to both individuals. Unlike the cases we have been discussing so far, such external entity is not an endurant (quality, mode, relator) but an *event*, namely, a person's birth. Moreover, comparative relations among objects are examples of internal and descriptive and relations similar to *married-with* are examples of external descriptive ones.

As discussed in depth in [6], there are important benefits, from a conceptual modeling point of view, in explicitly representing truthmakers via *relationships reification*, ranging from addressing ambiguity in *single-tuple* versus *multiple-tuple* cardinality constraints, clarifying the semantics of relations involving relations (e.g., relation subsetting, relation specialization, relation redefinition), modeling n-adic relations, etc. Guarino, Sales and Guizzardi [9], use these combinations devise a set of *truthmaking patterns* designed to properly represent truthmakers in all the cases where the relation merits reification, namely, all descriptive relations and some external non-descriptive ones. We explore these patterns in Section 4 incorporating them into the language as means to support ontology-driven conceptual modeling.

3 A Formal Theory of Relations

We present the first formalization of the aspects previously discussed. This formalization builds upon but significantly extends UFO's formalization for endurant types in [13][6] and serves as the foundational layer for Section 4.

Our formal theory is expressed in first-order modal logic QS5 with fixed domain of interpretation [4]. We omit the outermost necessity operator and universal quantifier, in case their scope takes the full formula. Assuming a fixed domain of interpretation, the elements of the domains are construed as *possibilia*, i.e., entities that exist at least in a possible world. UFO introduces then a non-logical *existence predicate* (`ex`) defined on the possible entities at issue (here `Thing`) (a1). By means of `ex`, we define the relation of *existential dependence* between two entities, $\texttt{ed}(x,y)$ (a2), and of existential independence, $\texttt{ind}(x,y)$, (a3). These axioms serve also the formalization of the *inherence* relation (a4)–(a7). A moment can be defined as an endurant that inheres in some endurant, which is the *bearer* of the moment (a10) (e.g. John's courage). Moreover, a moment cannot inhere in two separate individuals, (a8). By axiom (a10), the bearer of

[6] We only present an excerpt of the formalization here. The complete formalization is available at `https://github.com/diporello/UFO-Ontology-of-Relations/`.

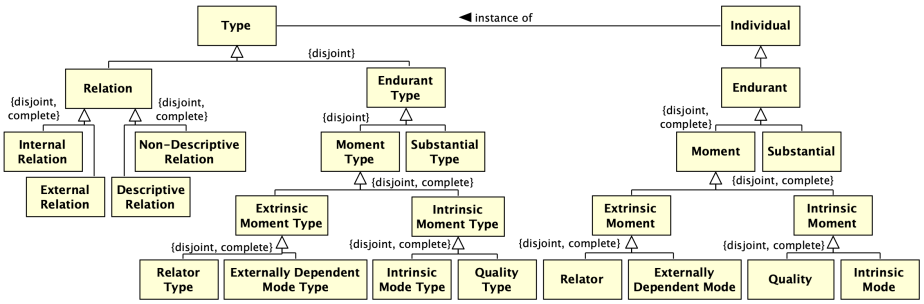

Fig. 2. Taxonomy of UFO Types.

a moment always exists and, by (a8), it is unique, thus the dependence is functional and we can introduce the function symbol $\beta(x)$ to indicate the unique *bearer* of a moment (d1). Moments are then partitioned into *intrinsic moments*, which existentially depend only on their bearer, and *extrinsic moments*, which depend on entities that are distinguished from the bearers (a11). Intrinsic moments are then divided into *qualities* (e.g. weight, length, color) in case the moment is measurable by a certain quality space (cf. [10]), and *intrinsic modes* (e.g. courage), which may not be measurable. *Extrinsic moments* are divided into *externally dependent modes* (edm) and *relators*. The former are moments that inhere in one entity and depends on another. We shall approach relators in the next paragraphs. Extrinsic moments do play a relational role between the entities on which they depend, cf. [10]. E.g. John's love for Mary inheres in John and existentially depends on Mary. We introduce the relation of external dependence, (a12).

a1 $\mathtt{ex}(x) \rightarrow \mathtt{Thing}(x)$ **a2** $\mathtt{ed}(x,y) \leftrightarrow \Box(\mathtt{ex}(x) \rightarrow \mathtt{ex}(y))$

a3 $\mathtt{ind}(x,y) \leftrightarrow \neg\mathtt{ed}(x,y) \wedge \neg\mathtt{ed}(y,x)$

a4 $\mathtt{inheresIn}(x,y) \rightarrow \mathtt{Moment}(x) \wedge \mathtt{Endurant}(y)$

a5 $\mathtt{inheresIn}(x,y) \rightarrow \mathtt{ed}(x,y)$ **a6** $\neg\mathtt{inheresIn}(x,x)$

a7 $\mathtt{inheresIn}(x,y) \wedge \mathtt{inheresIn}(y,z) \rightarrow \neg\mathtt{inheresIn}(x,z)$

a8 $\mathtt{inheresIn}(x,y) \wedge \mathtt{inheresIn}(x,z) \rightarrow y = z$

d1 $\beta(x) \leftrightarrow \iota y.\, \mathtt{inheresIn}(x,y)$ **a9** $\mathtt{inheresIn}(x,y) \rightarrow \neg\mathtt{inheresIn}(y,x)$

a10 $\mathtt{Moment}(x) \leftrightarrow \mathtt{Endurant}(x) \wedge \exists y\, \mathtt{inheresIn}(x,y)$

a11 $\mathtt{ExtrinsicMoment}(x) \leftrightarrow \mathtt{Moment}(x) \wedge \mathtt{ed}(x,y) \wedge \exists y.(\mathtt{ind}(y,\beta(x)))$

a12 $\mathtt{externallyDepends}(x,y) \leftrightarrow \mathtt{Moment}(x) \wedge \mathtt{ed}(x,y) \wedge \mathtt{ind}(y,\beta(x)))$

Finally, we assume a classical extensional mereology. For reason of space, we do not present the related axioms, relying on [15, 22]. We denote by Pxy and PPxy the relation of *part* and *proper part* (respectively).

We present now the theory of relations. We reify relations and we write $\langle x_1,\ldots,x_n\rangle :: r$ for the instantiation of an n-ary relation r by $x_1,\ldots,x_n$, cf. [16]. We limit ourselves to binary relations, the case of n-ary relations is a simple generalization. Hence, we can introduce a taxonomy of n-ary universals (i.e. relations), cf. Figure 2. The taxonomy

of relations is motivated by the specific truthmaking patterns, which are explicit in the right-hand part of axioms (a15) and (a16). This patterns indicate a necessary condition about the properties of the entities that are relevant to assess the relational statement at issue. By presenting the necessary conditions for the relational to hold (i.e. by $\rightarrow$), we are not committing to a characterization of the relational statement. For example, for an internal relation such as r = *heavier-than*, it is *necessary* for classifying $\langle x,y\rangle :: r$ that two qualities of these relata exists, namely, their weights. However, this is may not suffice, as we also need that the weight of x *is greater than* the weight of y. We approach this point in Section 4, where we characterize a particular subtype of internal relations.

To associate a relation (or, as we shall see, also a type) to the correct relevant properties, we assume a primitive relation of *derivation*, $\mathtt{der}(x,y)$. For instance, der associates a comparative relation such as r = *heavier-than* the *weight* qualities of the relata (and not e.g. the *colour* qualities).

An **internal relation**, (a15), holds *in virtue of intrinsic property* of the relata (e.g. *heavier-than* holds because of the weights of the relata). Defining the intrinsic properties of an entity is of course a difficult endeavour, cf. [9]. Here, we approximate, by assuming that intrinsic properties include types of intrinsic moments. Axiom (a13) does not exclude that we may list further intrinsic properties. Notice that the constraint about der is also required to define the relevant moment type that defines the intrinsic property. **External relations** are the non-internal ones, i.e. they are not reducible to relevant properties of the relata. As we shall discuss in the sequel, primitive relations are also construed here as external relations.

Descriptive relations are here restricted to mention moments of the relata, i.e. to simplify the presentation, we do not discuss moments that inhere the sum of the relata, cf. [9]. By (a16), descriptive relational statements may hold in two cases: *i*) in virtue of a pertinent extrinsic moment m that inheres in one of the relata and depends on the other (for external descriptive relations, e.g. *John admires Mary*) *or ii*) in virtue of the existence of pertinent intrinsic moments of the relata (for internal descriptive ones, e.g., John is *taller-than* Mary). Theorem (t1) indeed shows that, if r is descriptive and external, then there exists an extrinsic moment of one relatum that depends on the other.

a13 $\mathtt{Type}(p) \wedge \Box(x :: p \leftrightarrow \exists m,t(\mathtt{IntrinsicMoment}(m) \wedge \mathtt{IntrinsicMomentType}(t) \wedge m :: t \wedge \mathtt{inheresIn}(m,x)) \rightarrow \mathtt{IntrinsicProperty}(p)$

a14 $\mathtt{der}(x,y) \rightarrow (\mathtt{Relation}(x) \vee \mathtt{Type}(x)) \wedge (\mathtt{Relation}(y) \vee \mathtt{Type}(y))$

a15 $\mathtt{Internal}(r) \leftrightarrow \forall xy.(\Diamond\langle x,y\rangle :: r \rightarrow \exists pp'.(\mathtt{IntrinsicProperty}(p) \wedge \mathtt{IntrinsicProperty}(p') \wedge \mathtt{der}(r,p) \wedge \mathtt{der}(r,p') \wedge x :: p \wedge y :: p'))$

a16 $\mathtt{Descriptive}(r) \leftrightarrow \forall x_1x_2.(\Diamond\langle x_1,x_2\rangle :: r \rightarrow \exists z.(\mathtt{MomentType}(z) \wedge \mathtt{der}(r,z) \wedge \exists m.(\mathtt{ExtrinsicMoment}(m) \wedge m :: z \wedge \bigvee_{i,j\in\{1,2\}}^{i\neq j}(\mathtt{inheresIn}(m,x_i) \wedge \mathtt{ed}(m,x_j))) \vee \exists m_1m_2.((\bigwedge_{i\in\{1,2\}}(\mathtt{IntrinsicMoment}(m_i) \wedge m_i :: z \wedge \mathtt{inheresIn}(m_i,x_i))))))$

t1 $\mathtt{Descriptive}(r) \wedge \mathtt{External}(r) \wedge \Diamond\langle x_1,x_2\rangle :: r \rightarrow \exists xm.(\mathtt{MomentType}(x) \wedge \mathtt{ExtrinsicMoment}(m) \wedge \mathtt{der}(r,x) \wedge m :: x \wedge \bigvee_{i,j\in\{1,2\}}^{i\neq j}(\mathtt{inheresIn}(m,x_i) \wedge \mathtt{ed}(m,x_j))$

For internal relations, we have two cases of truthmaking. If they are also descriptive, we look for moments of the relata, e.g. the weight quality of the relata in a comparative statement between objects such as *John is heavier than Paul*. If they are not descriptive,

we search for intrinsic properties of the relata that are not moments. One example is the value of the weight quality in comparative statements between qualities as in *The weight of John is greater than the weight of Paul*, which is here understood as an intrinsic property of the relata but not a quality (a moment) of the relata.

For external non-descriptive relations, we have that there is no moment of the relata that is relevant to the truthmaking and also that there is no intrinsic properties of the relata to which we can reduce the relational statement. For this reason, external non-descriptive relations categorize our primitive undefined relations. For external descriptive relations, we have two cases of truthmaking. For *one-sided relations* (e.g. *John admires Mary*), the existence of the pertinent externally dependent mode suffices. For *double-sided relations* (*John is married to Mary*), a single externally dependent mode is not enough, we need the two modes inhering in both relata. That is, we need to introduce relators. *Relators* are formalized as mereological sums of externally dependent modes such that: they share the same foundation; they inhere in some entity; and, they existentially depend on another relatum, cf. [10]. We start by defining the *foundation* of an extrinsic moment as an event and we assume that the foundation is unique, cf (a17) and (a18). For reasons of space, we cannot fully discuss here the theory of events [7]. Axiom (a19) defines relators as objects that have at least two parts (cf. $\mathtt{P}mx$ and $\mathtt{P}nx$ in (a19)), which indeed are externally dependent modes that inhere some individual, share the same foundation, and depend on another individual.

a17 $\mathtt{foundedBy}(x,y) \rightarrow (\mathtt{ExtrinsicMoment}(x) \wedge \mathtt{Event}(y))$

a18 $\mathtt{ExtrinsicMoment}(x) \rightarrow \exists! y\ \mathtt{foundedBy}(x,y)$

a19 $\mathtt{Relator}(x) \leftrightarrow \exists mnyze.(\mathtt{Edm}(m) \wedge \mathtt{inheresIn}(m,y)\ \wedge\ \mathtt{Edm}(n) \wedge \mathtt{inheresIn}(n,z) \wedge \mathtt{P}mx \wedge \mathtt{P}nx \wedge m \neq n \wedge y \neq z \wedge \mathtt{foundedBy}(m,e) \wedge \mathtt{foundedBy}(n,e) \wedge \mathtt{ed}(m,z) \wedge \mathtt{ed}(n,y))$

a20 $\mathtt{mediates}(x,y) \leftrightarrow \mathtt{Relator}(x) \wedge \mathtt{Endurant}(y) \wedge \exists z.(\mathtt{Edm}(z) \wedge \mathtt{inheresIn}(z,y) \wedge \mathtt{P}zx)$

t2 $\mathtt{Relator}(x) \rightarrow \exists yz.(\mathtt{mediates}(x,y)\ \wedge \mathtt{mediates}(x,z) \wedge y \neq z)$

Mediation links a relator x and an individual y that the relator connects (a20). A relator is a particular type of moment, hence it has a unique bearer, which can be defined as the mereological sum of all the individuals mediated by the relator, cf. [15, 22]. By (a19) and (a20), a relator must connect at least two individuals (t2).

4 Towards a New UML Profile for Modeling Relations

OntoUML is an ODCM language that extends UML class diagrams by defining stereotypes that reflect UFO ontological distinctions into language constructs (e.g., classes and associations). As discussed previously, constructs decorated with OntoUML stereotypes carry a precise semantics grounded by UFO, and enriched by a set of *semantically motivated syntactical constraints* [2], reflecting UFO's axiomatization. In addition to ensuring ontological model consistency, the stereotyped constructs and constraints guide the modeler into addressing ontological issues concerning the subject domain. In particular, the OntoUML constructs for relations guide the modeler concerning the inclusion of truthmakers of domain relations in a model.

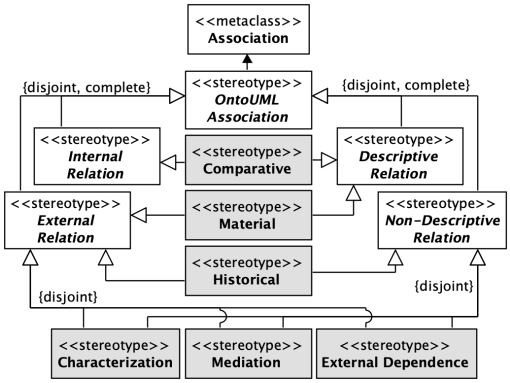

Fig. 3. OntoUML Profile for Relations.

The specification of OntoUML is presented as a UML profile (a *lightweight extension*) in Figure 3. All relation types are represented as stereotypes of UML associations. Stereotypes in gray are concrete, and, hence, are the only ones that appear in models. These stereotypes are discussed throughout this section, which concludes with a summary of the constraints governing their use (Table 1, reflecting the formalization). The stereotypes introduced here capture different types of domain relations that may hold between types of endurants. We shall recall the semantics for each of the stereotypes for endurant types used here, namely, «kind», «relatorKind», «modeKind», «qualityKind» and «role»[7]. The first four stereotypes in the list identify the ontological nature of the decorated type's instances and serve to mark the basic ontological categories instantiated by their instances. Types decorated by «kind» have object-like individuals (substantials) as instances (e.g., *Person*, *Car* or *Organization*); types decorated by «qualityKind» have qualities as instances(e.g., *Weight* or *Color*). Types decorated by «modeKind» have modes as instances (e.g., *Headache* or *Commitment*), including externally dependent modes. Types decorated by «relatorKind» have relators as instances (e.g., *Marriage* or *Enrollment*). These stereotypes are used to represent the *kinds* of entities in the domain, and capture *essential* properties of these entities, classifying them *necessarily*. For example, a *Person* is essentially so, although she can contingently be a *Student*, a *Wife*, a *Client*, an *Employee*, etc. Analogously, an *Enrollment* is essentially so, although it can contingently be a *suspended enrollment*, an *insured enrollment*, *grounds for visa application*, etc. The stereotype «role» decorates types that classify endurants of a given kind dynamically according to some relational property, e.g., the case for *Husband* and *Wife*, whose instances are instances of *Person* involved in a *married-with* relation (see Figure 4). Types stereotyped as «role» can specialize types decorated with any of the other mentioned class stereotypes.

The following OntoUML stereotypes for domains relations are defined: «characterization», «mediation», «external dependence», «comparative», «material», and «histor-

[7] The set of stereotypes for endurant types presented here is partial, but suffices for the interpretation of the discussed relations and examples. The complete list is drawn from [13].

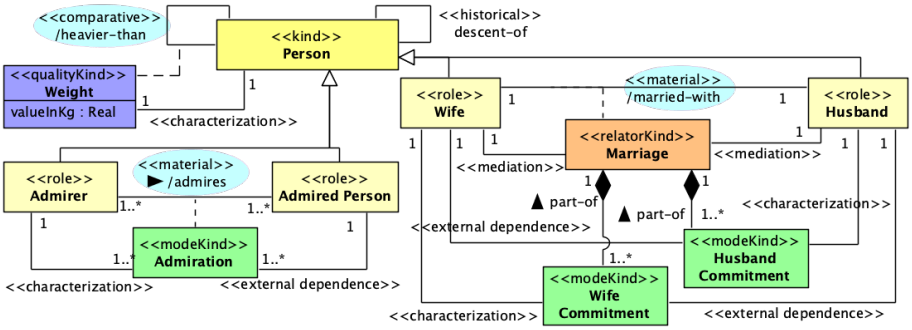

Fig. 4. OntoUML Patterns for the Reification of Relations.

ical». Their usage is exemplified in Figure 4. The «characterization», «mediation» and «external dependence» stereotypes decorate associations representing different sorts of existential dependencies, all external and non-descriptive. An association stereotyped with «characterization» connects a moment type (either a quality or a mode type) to the endurant type in which its instances *inhere*. An association stereotyped with «mediation» binds a relator type to each endurant type mediated by it. Finally, an association stereotyped with «external dependence» binds an externally dependent mode type to some endurant type on which its instances depend .

The «comparative» stereotype decorates associations representing comparative relations holding between endurant types. Comparative relations, such as *heavier-than*, are descriptive and internal, requiring the usage of a reification pattern to reveal their truthmakers. The truthmaker of a comparative relation is an equivalent relation holding between common qualities of the relata. For example, *heavier-than* holds between instances of *Person* whenever their qualities of *Weight* are related by a special relations (e.g., *greater-than* relation), i.e., the *heavier-than* relation is derived from a pair of weight qualities of the relata, which, in turn, are related by a *greater-than* relation (a internal and non-descriptive relation). Comparative relations are captured in the models by: (a) adding a derivation association (the dashed line in the model) connecting the comparative relation to the quality type of their truthmakers; (b) including a *derivation rule* in the model that strengthens axiom **(a15)** by including a condition representing the internal relation between the values of the appropriate qualities. In general, we have the following general *derivation pattern*: if a comparative relation C_R is derived from the quality type Q then $\langle x,y\rangle :: C_R \leftrightarrow (\exists q_x, q_y, r.\ (q_x :: Q \wedge q_y :: Q \wedge \texttt{inheresIn}(q_x,x) \wedge \texttt{inheresIn}(q_y,y) \wedge \texttt{Internal}(r) \wedge \neg\texttt{Descriptive}(r) \wedge \langle q_x,q_y\rangle :: r))$. In the example of Figure 4, this general pattern can take the following OCL form:

```
context Person::heavier-than: Set(Person)
derive: Person.allInstances()->select( p : Person |
                    self.weight.valueInKg > p.weight.valueInKg)
```

This pattern reveals not only the quality used as basis for the comparative relation, but also the way they relate to one another that makes true the comparison. With the

addition of «comparative» and this general *derivation rule pattern*, we deprecate the former «formal» stereotype.

The «material» stereotype decorates associations representing external descriptive relations, i.e., relations that hold *in virtue of* some relational endurant that is bound to the relata. The first kind of material relations acknowledged in OntoUML are those which truthmakers are relators, in which case the relator mediates all the relata and a derivation relation connects the material one to the relator, as it is for *married-with* and *Marriage*. In addition to relators, externally dependent modes can play the role of relational properties and truthmakers for one-sided material relations, in which case a mode type is connected through derivation to the relation and it inheres in one relatum (through *characterization*) and *externally depends* on the other, as it is for *admires* and *Admiration*. Externally dependent modes may also compose relators, requiring that they *inhere in* and *externally depend* on endurants mediated by the relator they are part of. Modes and relators capture the "life" of the relations derived by them, accounting for identity and properties of the relation that belongs not to the relata but to relation itself.

Lastly, the «historical» stereotype decorates associations representing historical relations. Historical relations are external and non-descriptive and, even though they may hold between endurants, their truthmakers are not endurants, but events (or compositions of events) responsible for the truth of the relation. This can be the case of *descent-of*, captured here as the relation holding between a person and each of his/her ancestors, all of whom participate in a chain of reproduction events. At this point, OntoUML does not officially account for the representation of events, thus, we include historical relations without a reification pattern for the inclusion of events as truthmakers. This is feature of our proposal to be revisited as soon as OntoUML incorporates primitives for the representation of events and event relations.

In addition to the rules presented throughout this section regarding the semantics of relations, their possible relata and truthmaking patterns, Table 1 collects additional constraints that emerge from our formalization. These constraints ensure the adherence to the truthmaking patterns discussed above. This profile is implemented as an extension for a UML CASE tool that incorporates the stereotypes for OntoUML 2.0 and syntactically verifies models for the language's constraints, informing the modeler of any violations or model incompleteness[8].

5 Final Considerations

We contributed to the ontological foundations of conceptual modeling by proposing a formal *ontological theory of relations*. We believe this theory makes an important contribution advancing the state of art in the field. Relations are one of conceptual modeling's most fundamental constructs. However, most existing foundational theories for conceptual modeling only recognize the most basic distinctions among the fundamental categories of relations. For example, the BWW ontology [25], which is the most used foundational ontology in ODCM [23], only countenances two types of properties, namely, *intrinsic and mutual properties*, and two types of relations, namely, *coupling*

[8] `https://github.com/nemo-ufes/OntoUML-2.0-for-Visual-Paradigm`

Table 1. OntoUML Constraints on External Descriptive Relations.

Constraints
From (a16) and (a19), associations decorated as «material» must have a derivation association towards a class decorated as «modeKind», for one-sided relations, and «relatorKind», for others.
From (a16) and (t1), classes decorated as «modeKind» and connected, through derivation, to some «material» relation must have a «characterization» relation towards one of the relata and an «external dependence» relation towards the other.
From (a19) and (t2), classes decorated as «relatorKind» and connected, through derivation, to some «material» relation must have a «mediation» relation towards each relata.
From (a19) and (a20), classes decorated «modeKind» and connected, through *part-of* relation, to some «relatorKind» must have a «characterization» relation towards one of the classes mediated (i.e., «mediation») by the relator.
From (a19), classes decorated «modeKind» and connected, through *part-of* relation, to some «relatorKind» must have a «external dependence» relation towards at least one of the classes mediated (i.e., «mediation») by the relator.
From (a15) and (a16), associations decorated as «comparative» must have a derivation association towards a class decorated as «quality».
From (a15) and (a16), classes decorated as «qualityKind» and connected, through derivation, to some «comparative» relation must have a «characterization» relation towards a class specialized by the relata or the relata themselves.

and non-coupling relations. As discussed in [11], the former distinction is analogous to our distinction between *intrinsic and extrinsic (i.e., externally dependent) moments*. Nevertheless, in our approach properties are instantiated, with several advantages (see in depth discussion in [10, 11]). Moreover, the BWW notion of mutual properties seems to conflate the (type-level counterpart of) our notions of externally dependent modes and relators. The latter distinction, as discussed in [10], is similar to the former UFO/OntoUML distinction between formal and material relations, which, as argued here, is insufficient to address subtle modeling requirements.

Our theory was developed to address a number of empirically elicited requirements, collected from observing the practice of the OntoUML community while using these notions to model a variety of domains (*claim to relevance*). Despite the empirical origin of these requirements, they are very much in line with the philosophical literature (*claim to ontological adequacy*). Additionally, following the same strategy as in [13], our formalization has been checked for its consistency using automated theorem provers (*claim to consistency*). Besides these foundations, we make a contribution to the practice of conceptual modeling by (re)designing a modeling profile based on this theory (following a well-tested approach to ontology-based language engineering [10]), and by providing a computational tool for model creation and verification according to this profile (*claim to realizability*). More broadly, the work presented here is part of a research program aimed at addressing a fuller evolution of UFO and OntoUML [13].

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