Representing Collectives and their Members in UML Conceptual Models: An Ontological Analysis

Giancarlo Guizzardi
Ontology and Conceptual Modeling Research Group (NEMO),
Federal University of Espírito Santo (UFES), Vitória (ES), Brazil
gguizzardi@inf.ufes.br

Abstract. In a series of publications, we have employed ontological theories and principles to evaluate and improve the quality of conceptual modeling grammars and models. In this article, we continue this work by conducting an ontological analysis to investigate the proper representation of types whose instances are *collectives*, as well as the representation of a specific part-whole relation involving them, namely, the *member-collective* relation. As a result, we provide an ontological interpretation for these notions, as well as modeling guidelines for their sound representation in conceptual modeling.

Keywords: representation of collectives and their members, ontological foundations for conceptual modeling, part-whole relations.

1 Introduction

In recent years, there has been a growing interest in the application of Foundational Ontologies, i.e., formal ontological theories in the philosophical sense, for providing real-world semantics for conceptual modeling languages, and theoretically sound foundations and methodological guidelines for evaluating and improving the individual models produced using these languages.

In a series of publications, we have successfully applied ontological theories and principles to analyze a number of fundamental conceptual modeling constructs ranging from Roles, Types and Taxonomic Structures, Relations, Attributes, Weak Entities and Datatypes, among others (e.g., [1-3]). In this article we continue this work by investigating a specific aspect of the representation of part-whole relations. In particular, we focus on the ontological analysis of *collectives* and of a specific part-whole relation involving them, namely, the *member-collective* relation.

Parthood is a relation of fundamental importance in a number of disciplines including cognitive science [4-6], linguistics [7-8], philosophical ontology [9-11] and conceptual modeling [1-3]. In ontology, a number of different theoretical systems have been proposed over time aiming to capture the formal semantics of parthood (the so-called *mereological relations*) [9,10]. In conceptual modeling, a number of so-called *secondary properties* have been used to further qualify these relations. These include distinctions which reflect different relations of ontological dependence, such as the distinction between *essential* and *mandatory* parthood [1,2]. Finally, in linguistic and cognitive science, there is a remarkable trend towards the definition of a typology of part-whole relations (the so-called *meronymic relations*) depending on the different types of entities they relate [7]. In general, these classifications include the following three types of relations: (i) *subquantity-quantity* (e.g., alcohol-wine, milk-milk shake): modeling parts of an amount of matter; (ii) *component-functional complex* (e.g., mitral valve-heart, engine-car): modeling aggregates of components, each of which contribute to the functionality of the whole; (iii) *member-collectives* (e.g., tree-forest, lion-pack, card-deck of cards, brick-pile of bricks).

This paper should then be seen as a companion to the publications in [2] and [3]. In the latter, we managed to precisely map the part-whole relation for quantities (the *subquantity-quantity* relation) to a particular mereological system. Moreover, in that paper, we managed to demonstrate which are the secondary properties implied by this relation. In a complementary manner, in [2], we exposed the limitations of classical mereology to model the part-whole relations between functional complexes (the *component – functional complex* relation). Additionally, we also managed to further qualify this relation in terms of the aforementioned secondary properties. The objective of this paper is to follow the same program for the case of the *member-collective* relation.

The remainder of this article is organized as follows. Section 2 reviews the theories put forth by classical mereology and discusses their limitations as theories of conceptual parthood. These limitations include the need for a *theory of (integral) wholes* to be considered in additional to a theory of parts. In section 3, we discuss collectives as integral wholes and present some modeling consequences of the view defended there. Moreover, we elaborate on some ontological properties of collectives that differentiate them not only from their sibling categories (quantities and functional complexes), but also from sets (in a set-theoretical sense). The latter aspect is of relevance since collectives as well as the member-collective relation are frequently taken to be identical to sets and the set membership relation, respectively. In section 4, we promote an ontological analysis of the *member-collective* relation, clarifying on how this relation stand w.r.t. to basic mereological properties (e.g., transitivity, weak supplementation, extensionality) as well as regarding the modal secondary property of essential parthood. As an additional result connected to this analysis, we outline a number of metamodeling constraints that can be used for the implementation of a UML modeling profile for representing collectives and their members in conceptual modeling. Section 5 presents some final considerations.

2 A Review of Formal Part-Whole Theories

2.1 Mereological Theories

In practically all philosophical theories of parts, the relation of (proper) parthood (symbolized as $<$) stands for a strict partial ordering, i.e., an asymmetric (2) and transitive relation (3), from which irreflexivity follows (1):

$$\forall x\ \neg(x < x) \tag{1}$$
$$\forall x,y\ (x < y) \rightarrow \neg(y < x) \tag{2}$$
$$\forall x,y,z\ (x < y) \wedge (y < z) \rightarrow (x < z) \tag{3}$$

These axioms amount to what is referred in the literature by the name of *Ground Mereology (M)*, which is the core of any theory of parts, i.e., the axioms (1-3) define the minimal (partial ordering) constrains that every relation must fulfill to be considered a parthood relation. Although necessary, these constraints are not sufficient, i.e., it is not the case any partial ordering qualifies as a parthood relation. Some authors [10], require an extra axiom termed the *weak supplementation principle* (4) as constitutive of the meaning of part and, hence, consider (1-3) plus (4) (the so-called *Minimal Mereology* (MM)) as the minimal constraints that a mereological theory should incorporate.

$$\forall x,y\ (y < x) \rightarrow \exists z\ (z < x) \wedge \neg \mathrm{overlap}(z,y) \tag{4}$$

An extension to MM has then been created by strengthening the supplementation principle represented by (4). In this system, (4) is thus replaced by the so-called stronger supplementation axiom[1]:

$$\forall x,y\ \neg(y \leq x) \rightarrow \exists z\ (z \leq y) \wedge \neg \mathrm{overlap}(z,x) \tag{5}$$

Formula (5) is named the *strong supplementation principle*, and the theory that incorporates (1-5) is named *Extensional Mereology (EM)*. A known consequence of the introduction of axiom (5) is that in EM, we have that two objects are identical iff they have the same (proper) parts, a mereological counterpart of the *extensionality principle* (of identity) in set theory.

A second way that MM has been extended is with the aim of providing a number of closure operations to the mereological domain. As discussed, for example, in [9], theories named *CMM (Closure Minimal Mereology)* and *CEM (Closure Extensional Mereology)* can be obtained by extending MM and EM with the operations of *Sum*, *Product*, *Difference* and *Complement*, which are the mereological counterparts of the operations of union, intersection, difference and complement in set theory. In particular, with an operation of sum (also termed *mereological fusion*), one can create an entity which is the so-called *mereological sum* of a number of individuals.

2.2 Problems with Mereology as a Theory of Conceptual Parts

Mereology has shown itself useful for many purposes in mathematics and philosophy [9,10]. Moreover, it provides a sound formal basis for the analysis and representation of the relations between parts and wholes regardless of their specific nature. However, as pointed out by [4,5] (among other authors), it contains many problems that make it hard to directly apply it as a theory of conceptual parts. As it shall become clear in the discussion that follows, on one hand the theory is too strong, postulating constraints that cannot be accepted to hold generally for part-whole relations on the conceptual

[1]The improper parthood relation ($\leq$) in this formula can be defined as $(x \leq y) =_{def} (x < y) \vee (x = y)$.

level. On the other hand, it is too weak to characterize the distinctions that mark the different types of conceptual part-whole relations.

A problem with ground mereology is the postulation of unrestricted transitivity of parthood. As discussed in depth in the literature [2,8], there many cases in which transitivity fails. In general, in conceptual modeling, part-whole relations have been established as non-transitive, i.e., transitive in certain cases and intransitive in others.

The problem with extensional mereologies from a conceptual point of view arises from the introduction of the strong supplementation principle (5) which states that objects are completely defined by their parts. If an entity is identical to the mereological sum of its parts, thus, changing any of its parts changes the identity of that entity. Ergo, an entity cannot exist without each of its parts, which is the same as saying that all its parts are *essential parts*. Essential parthood can be defined as a case of *existential dependence* between individuals, i.e., x is an *essential part* of y iff y cannot possibly exist without having that specific individual x as part [1]. A stereotypical example of an essential part of a car is its chassis, since that specific car cannot exist without that specific chassis (changing the chassis legally changes the identity of the car). As discussed in depth in [1], essential parthood plays a fundamental role in conceptual modeling. However, while some parts of objects represented in conceptual models are essential, not all of them are so. The failure to acknowledge that can be generalized as the failure of classical mereological theories to take into account the different roles that parts play within the whole. As discussed in [1,3], a conceptual theory of parthood should also countenance a *theory of wholes*, in which the relations that tie the parts of a whole together are also considered.

From a conceptual point of view, the problem with the theory of General (Classical) Extensional Mereology is related to the existence of a mereological sum (or fusion) for any arbitrary non-empty (but non-necessarily finite) set of entities. Just as in set theory where one can create a set containing arbitrary members, in GEM one can create a new object by summing up individuals that can even belong to different ontological categories. For example, in GEM, the individual created by the sum of Noam Chomsky's left foot, the first act of Puccini's Turandot and the number 3, is an entity considered as legitimate as any other. As argued by [4], humans only accept the summation of entities if the resulting mereological sum plays some role in their conceptual schemes. To use an example: the sum of a frame, a piece of electrical equipment and a bulb constitutes an integral whole that is considered meaningful to our conceptual classification system. For this reason, this sum deserves a specific concept in cognition and name in human language. The same does not hold for the sum of bulb and the lamp's base. Once more, we advocate that a theory of conceptual parthood must also comprise a theory of wholes.

According to Simons [10], the difference between purely formal mereological sums and, what he terms, *integral wholes* is an ontological one, which can be understood by comparing their existence conditions. For sums, these conditions are minimal: the sum exists just as the constituent parts exist. By contrast, for an integral whole (composed of the same parts of the corresponding sum) to exist, a further *unifying condition* among the constituent parts must be fulfilled. A unifying condition or relation can be used to define a closure system in the following manner. A set B is a closure system under the relation R, or simply, R-closure system iff

$$\mathbf{cs}\ \langle \mathbf{R}\rangle\ \mathbf{B} =_{\mathbf{def}} (\mathbf{cl}\ \langle \mathbf{R}\rangle\ \mathbf{B}) \wedge (\mathbf{con}\ \langle \mathbf{R}\rangle\ \mathbf{B}) \qquad \mathbf{(6)}$$

where **(cl ⟨R⟩ B)** means that the set B is closed under R (R-Closed) and **(con ⟨R⟩ B)** means that the set B is connected under R (R-Connected). R-Closed and R-Connected are then defined as:

$$\mathbf{cl}\ \langle \mathbf{R}\rangle\ \mathbf{B} =_{\mathbf{def}} \forall \mathbf{x}\ (\mathbf{x}\in\mathbf{B}) \rightarrow ((\forall \mathbf{y}\ \mathbf{R}(\mathbf{x},\mathbf{y}) \vee \mathbf{R}(\mathbf{y},\mathbf{x}) \rightarrow (\mathbf{y}\in\mathbf{B})) \qquad \mathbf{(7)}$$

$$\mathbf{con}\ \langle \mathbf{R}\rangle\ \mathbf{B} =_{\mathbf{def}} \forall \mathbf{x}\ (\mathbf{x}\in\mathbf{B}) \rightarrow (\forall \mathbf{y}\ (\mathbf{y}\in\mathbf{B}) \rightarrow (\mathbf{R}(\mathbf{x},\mathbf{y}) \vee \mathbf{R}(\mathbf{y},\mathbf{x})) \qquad \mathbf{(8)}$$

An integral whole is then defined as an object whose parts form a closure system induced by what Simons terms a *unifying* (or *characterizing*) *relation* R.

3 What are Collectives?

In an orthogonal direction to the mereological theories just discussed, there are foundational theories in linguistic and cognitive science developed to offer a characterization of the relation of parthood. A classical work in this direction is the one of Winston, Chaffin and Herrmann [7] (henceforth WCH). WCH propose an account of the notion of parthood by elaborating on different types of part-whole relations depending on different ways that a part can be related to a whole. These distinctions have proven themselves fundamental for the development of a general parthood theory for conceptual modeling. Moreover, as it has been shown in a number of publications, issues such as transitivity, essentiality of parts, as well as the definition of characterizing relations, are not orthogonal to these fundamental distinctions. For instance, [3] demonstrates that: (i) the *subquantity-quantity* relation obeys the axiomatization of the so-called Extensional Mereology (EM), i.e., it is an irreflexive, anti-symmetric and transitive relation; (ii) all subquantities of a quantity are essential parts of it; (iii) quantities are unified by a relation of *topological-maximal-self-connectedness*.

According to WCH, the main distinction between collections and quantities is that the latter but not the former are said to be *homeomeros* wholes. In simple terms, homeorosity means that the entity at hand is composed solely of parts of the same type (*homo*=same, *mereos* = part). The fact that quantities are homeomerous (e.g., all subportions of wine are still wine) causes a problem for their representation (and the representation of relationships involving them) in conceptual modeling. In order to illustrate this, we use the example depicted in figure 1.a below. In this specification, the idea is to represent that a certain portion of wine is composed of all subportions of wine belonging to a certain vintage, and that a wine tank can store several portions of wine (perhaps an *assemblage* of different vintages). However, since Wine is homeomeros and infinitely divisable in subportions of the same type, we have that if a Wine portion x has as part a subportion y then it also has as parts all the subparts of y [3]. Likewise, a wine tank storing two different "portions of wine" actually stores all the subparts of these two portions, i.e., it stores infinite portions of wine. In other words, maximum cardinality relations involving quantities cannot be specified in a finite manner. As discussed, for instance in [3], *finite satisfiability* is a fundamental requirement for conceptual models which are intended to be used in areas such as Databases and Software Engineering. This feature of quantities, thus, requires a special treatment so that they can be property modeled in structural conceptual models, and one that does not take quantities to be simply mereological sums of subportions of the same kind [3].

Figure 1 UML Representations of a Quantity (a-left) and a Collective (b-right) with their respective parts

As correctly defined by WCH, collectives are not homeomeros. They are composed of subparts parts that are not of the same kind (e.g., a tree is not forest). Moreover, they are also not infinitely divisible. As a consequence, a representation of a collection as a mereological sum of entities (analogous to a set of entities) does not lead to the same complications as for the case of quantities. Take, for instance, the example depicted in figure 1.b, which represents a situation analogous to that of figure 1.a. In contrast with from the former case, there is no longer the danger of an infinite regress or the impossibility for specifying finite cardinality constraints. In figure 1.b, the usual maximum cardinality of “many” can be used to express that group of people has as parts possibly many other groups of people and that a guide is responsible for possibly many groups of people.

Nonetheless, in many examples (such as this one), the model of figure 1.b implies a somewhat counterintuitive reading. In general, the intended idea is to express that, for instance, John as a guide, is responsible for the group formed by {Paul, Marc, Lisa} and for the other group formed by {Richard, Tom}. The intention is not to express that John is responsible for the groups {Paul, Marc, Lisa}, {Paul, Marc}, {Marc, Lisa}, {Paul, Lisa}, and {Richard, Tom}, i.e., that being responsible for the group {Paul, Marc, Lisa}, John should be responsible for all its subgroups. A simple solution to this problem is to consider groups of as maximal sums, i.e., groups that are not parts of any other groups. In this case, depicted in figure 2, the cardinality constraints acquire a different meaning and it is no longer possible to say that a group of people is composed of other groups of people.

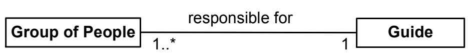

Figure 2. Representation of Collections as Maximal Sums

This solution is similar to taking the meaning of a quantity K to be that of a maximally-self-connected-portion of K [3]. However, in the case of collections, topological connection cannot be used as a *unifying or characterizing relation* to form an integral whole, since collections can easily be spatially scattered. Nonetheless, another type of connection (e.g., social) should always be found. A question begging issue at this point is: why does it seem to be conceptually relevant to find *connection relations* leading to (maximal) collections? As discussed in the previous section, collections taken as arbitrary sums of entities make little cognitive sense: we are not interested in the sum of a light bulb, the North Sea, the number 3 and Aida's second act. Instead, we are interested in aggregations of individuals that have a purpose for some cognitive task. So, we require all collectives in our system to form closure systems unified under a proper characterizing relation. For example, a group of people of interest can be composed by all those people that are attending a certain museum exhibition at a certain time, or all the people under 18 which have that been exposed to some disease. Now, by definition, a closure system is maximal (see formula (6)), thus, there can be no group of people in this same sense that is part of another group of people (i.e., another integral whole unified by the same relation).

Some authors (e.g., [5]) propose that the difference between a collection and a functional complex is that whilst the former has a *uniform* structure, the latter has a *heterogeneous* and *complex* one. We propose to rephrase this statement in other terms. In a collection, all member parts play the same role type. For example, all trees in a forest can be said to play the role of a forest member. In complexes, conversely, a variety of roles can be played by different components. For example, if all ships of a fleet are conceptualized as playing solely the role of "member of a fleet" then it can be said to be a collection. Contrariwise, if this role is further specialized in "leading ship", "defense ship", "storage ship" and so forth, the fleet must be conceived as a functional complex. In summary, collections as *integral wholes* (i.e., in a sense that appeals to cognition and common sense conceptual tasks) can be seen as limit cases of Gerlst and Pribbenow's *functional complex* [5], in which parts play one single role forming a uniform structure.

Finally, we would like to call attention to the fact that collectives are not sets and, thus, the *member-collective* relation is not the same as the *membership* ($\in$). Firstly, collectives and sets belong to different ontological categories: the former are concrete entities that have spatiotemporal qualities; the latter, in contrast, are abstract entities that are outside space and time and that bear no causal relation to concrete entities [1]. Secondly, unlike sets, collectives do not necessarily obey an extensional principle of identity, i.e., it is not the case that a collective is completely defined by the sum of its members. We take that some collectives can be considered extensional by certain conceptualizations, however, we also acknowledge the existence of *intentional collectives* obeying non-extensional principles of identity [6]. Thirdly, collectives are integral whole unified by proper characterizing relations; sets (as mereological sums), in contrast, can be simply postulated by enumerating its members (or parts). This feature of the latter is named *ontological extravagance* and it is a feature to be ruled out from an ontological system [9]. Finally, we do not admit the existence of empty or unitary collectives, contrary to set theory which admits both the empty set $\varnothing$ and sets with a unique element. As a consequence, we eliminate a feature of set theory named *ontological exuberance* [9]. Ontological exuberance refers to the feature of some formal systems that allows for the creation of a multitude of entities without differentiation in content. For instance, in set theory, the elements a, $\{a\}$, $\{\{a\}\}$, {{{a}}}, {…{{{a}}}…} are all considered to be distinct entities. We shall return to some of these points in the next section.

4 The Member-Collection Relation

According to [8], classical semantic analysis of plurals and groups distinguish between *atomic entities*, which can be *singular* or *collectives*, and *plural entities*. From a linguistic point of view, the *member-collection* relation is considered to be one that holds between an *atomic entity* (e.g., John, the deck of cards) and either a *plural* (e.g., {John, Marcus}) *or a collective term* (e.g., the children of Joseph, the collection of antique decks).

Before we can continue, a formal qualification of this notion of atomicity is required. Suppose an integral whole W unified under a relation R. By using this characterizing relation R, we can then define a composition relation $<_R$ such that (x $<_R$ W) iff: (i) there is a set B such that **cs ⟨R⟩ B**; (iii) (x < W) and (x ∈ B). Intuitively, this relation captures the idea that there is indeed a genuine connection between a part x and the whole W, as opposed to a merely formal one. Now, one important thing to highlight is that if (x $<_R$ W) then there is no y such that (y $<_R$ x). In other words, the closure set defined by relation R are the R-atoms of W. This is because, the whole W unified under R is maximal under this relation, by the definition of an R-closure system. The fact that no R-part of W can be unified under the same relation R, of course, does not imply that these R-parts need to be atomic in an absolute sense. In fact, given an element x such that (x $<_R$ W), x itself can be an integral wholes unified by a different relation R'. However, it should be clear by now that the sets of R'-atoms of x and the set of R-atoms of W (of which x is a member) are disjoint.

The *member-collective* (symbolized as *M(part,whole)*) is an example of a relation that takes place between an atom under relation R and an integral whole unified under that relation. Following the above discussion, we have that these relations are never transitive, i.e., they are intransitive. Thus, if M(x,W) then x is atomic for W, and if we have M(y,x), we also have necessarily that ¬M(y,W). In other words, for the case of the member-collective relation, to say that a member must be a *singular* entity coincides with this entity being an *atom* in the sense just discussed, i.e., an atom w.r.t. to a characterizing relation unifying that specific whole.

An example of a member-collection relation is the following: *An individual x is a member of a club C (collective) and C is a member of an International body of Clubs C′(collective). However, it does not follow that x is member of C′, since C′ only has clubs as members, not individuals*. This situation is depicted in figure 3, in which we decorate the standard UML symbol for aggregation with an M to represent a member-collective parthood relation.

Regarding the weak supplementation axiom, some authors claim that this axiom would be too hard a constraint to be imposed to the member-collective relation [11]. From a formal point of view, this view implies that we accept reflexive characterizing relations for collectives as integral wholes. Such an approach seems at first to be somehow afforded by common sense. For instance, we can conceive a book of poems composed of a single poem, a CD composed of a single track, a purchase order composed of single order item, or a journal issue composed by a single article. Now, are there disadvantages to such an approach? We can foresee two of them.

Firstly, abandoning weak supplementation would set this relation apart from all the other types of parthood relations, since this axiom (considered to be constitutive of the very meaning of part) is assumed by the relations of *component-functional complex* [2] and *subquantity-quantity* [3]. Secondly, this choice opens the possibility for the creation of collectives with one single member. But what then would be the difference between John, {John}, {{John}}, {…{{John}}…}, etc? If entities such as these are generally adopted, then our system can face the objection of *ontological exuberance*, and we should be reminded that avoiding this feature was one of the motivations of mereology in the first place [9]. Of course, one can state that we should require characterizing relations to be informative, i.e., it is not the case that any formal predicate should count as a characterizing relation. But, if we take singletone properties to count as characterizing relations, we need to be much more careful to differentiate which properties should count as informative and which should not. Given these two reasons, we adopt in this paper the view that weak supplementation should be part of the axiomatization of the member-collective relation. This, obviously, does not imply that we cannot have single-track CD's or single-article journal issues. Following [1], in these cases, we consider the relation between, for instance, the tracks and the CDs to be a relation of *constitution* as opposed to one of parthood. Relations of constitution abound in ontology. An example is the relation between a marble statue and the (single portion of marble) that constitutes it [1].

The discussion in this section is summarized as follows: (i) Member-collective is an irreflexive, anti-symmetric but *intransitive* relation. Moreover, it obeys the weak supplementation axiom; (ii) A member x of a collective W is atomic w.r.t. the collective. This means that for if an entity y is part of x then y is not a member of W; (iii) Collectives are not necessarily extensional entities. But, if there is a member of a collective W which is essential to W then all other members of W are essential to it.

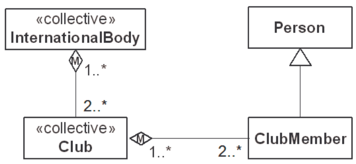

Figure 3. Examples of member/collection part-whole relations

5 Final Considerations

The development of suitable foundational theories is an important step towards the definition of precise real-world semantics and sound methodological principles for conceptual modeling languages. This article complements a sequence of papers that aim at addressing the three fundamental types of wholes prescribed by theories in linguistics and cognitive sciences, namely, functional complexes, quantities, and collectives. The first of these roughly correspond to our common sense notion of object and, hence, the standard interpretation of objects (or entities) in the conceptual modeling literature is that of a functional complex. The latter two categories, in contrast, have traditionally been neglected both in conceptual modeling as well as in the ontological analyzes of conceptual modeling grammars.

In this paper, we conduct an ontological analysis to investigate the proper representation of types whose instances are collectives, as well as the representation of an important parthood relation involving them. As result, we are able to provide a sound ontological interpretation for this notion, as well as modeling guidelines for the proper representation of collectives in conceptual modeling. In addition, we have managed to provide a precise qualification for the relation of *member-collective* w.r.t. to both classical mereological properties (e.g., transitivity, weak supplementation, extensionality) as well as the modal secondary property of essentiality of parts. Finally, the results advanced here contribute to the definition of concrete engineering tools for the practice of conceptual modeling. In particular, the metamodel extensions and associated constraints outlined here have been implemented in a Model-Driven Editor using available UML metamodeling tools [12].

Acknowledgement: This research is funded by the Brazilian Research Funding Agencies FAPES (Grant# 45444080/09) and CNPq (Grant# 481906/2009-6).

References

[1] Guizzardi, G., Ontological Foundations for Structural Conceptual Models, Telematica Institute Fundamental Research Series, The Netherlands (2005).

[2] Guizzardi, G. The Problem of Transitivity of Part-Whole Relations in Conceptual Modeling Revisited, In: P. van Eck, J. Gordijn and R. Wieringa (eds.), CAISE 2009, LNCS, vol. 5565, pp.94-109, Springer, Berlin (2009).

[3] Guizzardi, G., On the Representation of Quantities and their Parts in Conceptual Modeling, Frontiers in Artificial Intelligence, Vol. 209, pp.103-116, IOS Press, Amsterdam (2010).

[4] Pribbenow, S. Meronymic Relationships: From Classical Mereology to Complex Part-Whole Relations, The Semantics of Relationships, R.Green and C. A. Bean (eds.), pp.35-50, Kluwer, Dordretch (2002).

[5] Gerstl, P. and Pribbenow, S., Midwinters, End Games, and Bodyparts. A Classification of Part-Whole Relations, International Journal of Human-Computer Studies, Vol. 43, pp. 865-889 (1995).

[6] Botazzi, E. et al., From Collective Intentionality to Intentional Collectives: An Ontological Perspective, Cognitive Systems Research, Vol. 7, Issues 2-3, pp.192-208 (2006).

[7] Winston M.E.; Chaffin R.; Herrman D. A taxonomy of part-whole relations, Cognitive Science, Vol.11, no. 4, pp.417-444 (1987).

[8] Vieu, L.; Aurnague, M. Part-of Relations, Functionality and Dependence, Categorization of Spatial Entities in Language and Cognition, John Benjamins, Amsterdam (2007).

[9] Varzi. A.C., Parts, wholes, and part-whole relations: The prospects of mereotopology. Journal of Data and Knowledge Engineering, Vol. 20, pp. 259–286 (1996).

[10] Simons, P.M. Parts. An Essay in Ontology, Clarendon Press, Oxford (1987).

[11] Bittner, R., Donelly, M., Smith, B., Individuals, Universals, Collections: On the Foundational Relations of Ontology, Frontiers in Artificial Intelligence, Vol. 114, IOS Press, Amsterdam (2010).

[12] Benevides, A.B.; Guizzardi, G. A Model-Based Tool for Conceptual Modeling and Domain Ontology Engineering in OntoUML, In: J. Filipe and J. Cordeiro, ICEIS 2009, LNBIP, Vol. 24, pp.528-538, Springer, Berlin (2009).