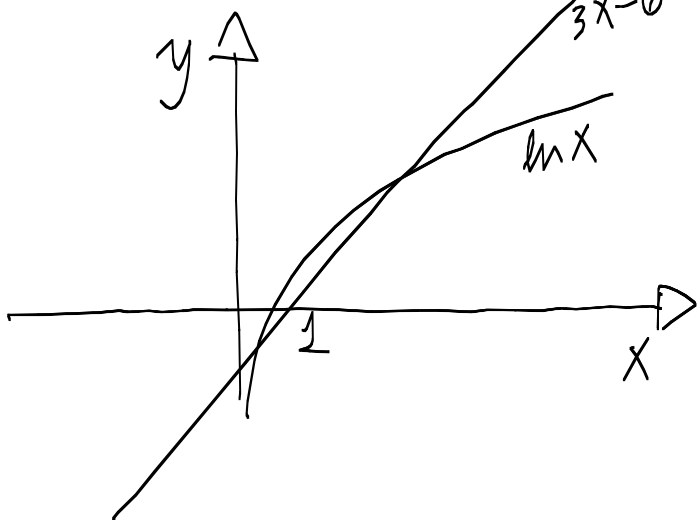
Note Title 6/14/2009

2ª Prova de Cálculo Numérico - 09/01

1) a) $f(x) = -3x + 6 + \ln x = 0$

$\ln x = 3x - 6$



$\xi_1 \in (0,1)$
$\xi_2 \in (1,4)$

	1	4
	+	−

ξ_2

b) $[a,b] = [1,4]$ $\quad f(a) > 0$, $f(b) < 0$

1) $x = 2.5 \quad f(x) < 0$

$[a,b] = [1, 2.5]$ $\quad f(a) > 0$, $f(b) < 0$

2) $x = 1.75 \quad f(x) > 0$

$[a,b] = [1.75, 2.5]$

$$K > \frac{\log(b-a) - \log \varepsilon}{\log 2}$$

$a = 1$
$b = 4$
$\varepsilon = 10^{-5}$

$K > 18.19 \Rightarrow K = 19$

c) $3x + 6 + \ln x = 0$

$$x = \frac{1}{3}\ln x - 2 = \varphi(x)$$

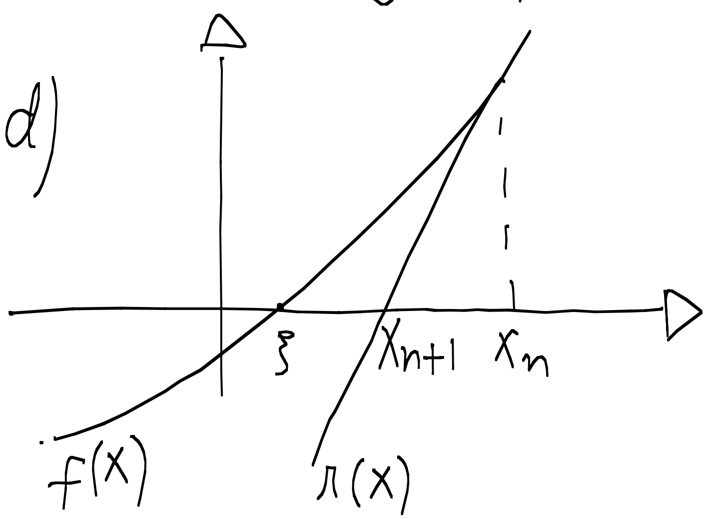
$$|\varphi'(x)| = \left|\frac{1}{3x}\right| < 1 \Rightarrow |3x| > 1$$

$$|x| > 1/3$$

$$x > 0.333 \qquad f(x) > 0$$

$\Rightarrow$ Converge p/ $\xi \in [1/3, 4]$

d)



$r(x)$: reta que passa por (x_n, y_n) e tem inclinação $k = f'(x_n)$

$$\frac{r(x) - f(x_n)}{x - x_n} = f'(x_n)$$

O ponto $(x_{n+1}, 0)$ pertence a $r(x)$

$$\Rightarrow \frac{0 - f(x_n)}{x_{n+1} - x_n} = f'(x_n)$$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

e) $a = 1 \qquad fa = 3$
$b = 4 \qquad fb = -4.6137$

$|fa| < |fb| \Rightarrow a = 4 \quad fa = -4.6137$
$b = 1 \quad fb = 3$

$$deltax = \frac{-fb}{fb - fa}(b-a) = 1.1821$$

$x = b + deltax = 2.1821 \qquad fx = 0.2340$

$|deltax| > \varepsilon \qquad |fx| > \varepsilon$

$a = 1 \qquad fa = 3$
$b = 2.1821 \qquad fb = 0.2340$

$deltax = 0.1$

$x = 2.2821 \qquad fx = 0.02120$

$|deltax| > \varepsilon \qquad |fx| > \varepsilon$

f) $\lim_{n \to \infty} |e_{n+1}| = K\,|e_n|^{\alpha} \qquad \alpha =$ ordem de convergência

$\alpha = 1 \quad K < 1 \quad$ MIL
$\alpha = 2 \quad$ Newton $\leftarrow$ converge mais rápido
$\alpha \simeq 1.6 \quad$ secante

2) a) $\begin{cases} \frac{dy}{dx} = xe^{-xy} \\ y(2) = 1 \end{cases}$ $h = 0.15$

$$y_{n+1} = y_n + \frac{0.15}{2}(K_1 + K_2)$$

$$\begin{cases} K_1 = f(x_n, y_n) \\ K_2 = f(x_n + 0.15, y_n + 0.15K_1) \end{cases}$$

$x_0 = 2$ $y_0 = 1$

$K_1 = 0.2707$
$K_2 = 0.2295$

$\Rightarrow y_1 = 1.0375 \approx y(2.15)$

$x_1 = 2.15$ $y_1 = 1.0375$

$K_1 = 0.2310$
$K_2 = 0.1953$

$\Rightarrow y_2 = 1.0695 \approx y(2.30)$

err local $= O(h^3)$

err global $= O(h^2)$

b) $$\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y^2 = 0$$

$$y_1 = y \qquad \begin{cases} y_1' = y_2 \\ y_2' = x \cdot y_2 - y_1^2 \end{cases} \quad \begin{matrix} y_1(1) = 1 \\ y_2(1) = 2 \end{matrix}$$
$$y_2 = y'$$

$$\begin{cases} \frac{dY}{dX} = F(X, Y) \\ Y(1) = Y_0 \end{cases} \qquad \begin{matrix} X_0 = 1 \\ h = 0.2 \end{matrix}$$

$$Y_{n+1} = Y_n + 0.2F(X_n, Y_n)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{n+1} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_n + 0.2\begin{bmatrix} y_2 \\ Xy_2 - y_1^2 \end{bmatrix}_n$$

$$X_0 = 1 \quad Y_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0.2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 2.2 \end{bmatrix}$$

$$X_1 = 1.2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_2 = \begin{bmatrix} 1.4 \\ 2.2 \end{bmatrix} + 0.2\begin{bmatrix} 2.2 \\ 0.68 \end{bmatrix} = \begin{bmatrix} 1.84 \\ 2.336 \end{bmatrix}$$

$$\frac{dy}{dx}(1.4) \approx 2.336$$