Um Pequeno Conjunto de Fórmulas (Possivelmente Úteis)

Berilhes Borges Garcia
Universidade Federal do Espírito Santo

January 23, 2003

1 Exponenciais

Para todo $a \neq 0$, m, e n, nós temos que as seguintes identidades são verdadeiras:

$$\begin{aligned} a^0 &= 1 \\ a^1 &= a \\ a^{-1} &= \frac{1}{a} \\ (a^m)^n &= (a^n)^m \\ a^m a^n &= a^{n+m} \end{aligned} \tag{1}$$

$$\lim_{n\to\infty} \frac{n^b}{a^n} = 0 \tag{2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{i=0}^{\infty} \frac{x^i}{i!} \tag{3}$$

2 Logaritmos

Para todo $a > 0, b > 0, c > 0$ reais, e n :

$$\begin{aligned} a &= b^{\log_b a} \\ \log_c(ab) &= \log_c a + \log_c b \\ \log_b a &= \frac{\log_c a}{\log_c a} \\ \log_b(1/a) &= -\log_b a \\ \log_b a &= \frac{1}{\log_a b} \\ a^{\log_b n} &= n^{\log_b a} \\ a^{\log_{1/a} n} &= \frac{1}{n} \end{aligned} \tag{4}$$

$$\lim_{n\to\infty} \frac{\lg^b n}{n^a} = 0 \tag{5}$$

3 Fatoriais

$$n! = \begin{cases} 1 & \text{se } n = 0 \\ n \cdot (n-1)! & \text{se } n > 0 \end{cases} \tag{6}$$

Aproximação de Stirling:

$$n! = \sqrt[2]{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right) \tag{7}$$

4 Funções Piso e Teto

$$\begin{aligned} \lfloor x \rfloor &= \text{ maior inteiro menor ou igual à } x \\ \lceil x \rceil &= \text{ menor inteiro maior ou igual à } x \end{aligned} \tag{8}$$

$$\begin{aligned} \lfloor x \rfloor = n &\Leftrightarrow n \leq x < n+1 \\ \lfloor x \rfloor = n &\Leftrightarrow x-1 < n \leq x \\ \lceil x \rceil = n &\Leftrightarrow n-1 < x \leq n \\ \lceil x \rceil = n &\Leftrightarrow x \leq n < x+1 \end{aligned} \tag{9}$$

$$\begin{aligned} \lfloor n/2 \rfloor + \lceil n/2 \rceil &= n \\ \lceil \lceil n/a \rceil / b \rceil &= \lceil n/ab \rceil \\ \lfloor \lfloor n/a \rfloor / b \rfloor &= \lfloor n/ab \rfloor \end{aligned} \tag{10}$$

5 Séries

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{1}{2} n(n+1) \tag{11}$$

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{x^{n+1}-1}{x-1} \tag{12}$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad \text{se } |x| < 1 \tag{13}$$

$$\sum_{k=1}^{n} \frac{1}{k} = \ln n + O(1) \tag{14}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \tag{15}$$