Adaptive Control for Time Step Selection in Finite Element Simulation of Coupled Viscous Flow and Heat Transfer

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ABSTRACT

Summary A decoupled finite element formulation with adaptive feedback control for time step selection has been developed for 2D viscous flow problems involving heat transfer and surface tension effects. The finite element flow formulation is based on a penalty Galerkin method and the heat equation utilizes a Galerkin approach. Representative Rayleigh-Benard and Marangoni flow calculations are presented, and the efficiency of the present time step scheme is examined and compared with other time-stepping strategy.

Coupled viscous flow and heat transfer computations are of great interest in studying pattern formation in hydrodynamical systems. Practical applications include, for example, pattern formation during solidification, welding in manufacturing processes and growth phenomena to defect fracture and crack propagation. Rayleigh-Benard-Marangoni problems become very popular as prototypes of complex behavior where nonlinear theories of pattern formation may be tested.

To develop effective algorithms capable of high resolution transient flow and heat transfer computations, we need improved techniques such as adaptive grid refinement and coarsing. For example, the use of an adaptive timestepping strategy can provide stable accurate transient (or steady-state) solutions more efficiently. Our main objective in the present work is the utilization of a PID control algorithm for timestep selection [1] in conjunction with finite element analysis to solve coupled problems such as Rayleigh-Benard-Marangoni flows.

We consider the transient flow of viscous incompressible fluid as described by the Navier-Stokes equations coupled to the heat transfer equation. Accordingly, the time-dependent form of the governing equations are

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} + \boldsymbol{\nabla} p = -\beta(T - T_0)\mathbf{g} \quad (1)$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0 \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T - \nabla\cdot(\alpha\nabla T) = 0 \quad (3)$$

where **u** is the velocity, p is the pressure, T is the temperature, T_0 is the reference temperature, ν is the kinematic viscosity, β is the thermal coefficient, **g** is the gravity vector, and α is the thermal diffusivity. Boundary conditions and initial conditions complete the mathematical statement of the problem. Thermocapillary stresses appear on a free surface due to thermal gradients.

The finite element flow formulation is based on a penalty Galerkin method to enforce the incompressibility constraint. Introducing a finite element discretization and a basis functions, the resulting semi-discrete ODE system for the velocities is integrate implicitly using a Crank-Nicolson scheme. At each timestep we solve the nonlinear system iteratively using successive approximations with frontal elimination of the resulting linear systems. The heat equation utilizes a Galerkin approach, and the semi-discrete ODE system is also integrated using a Crank-Nicolson scheme, where at each time step the resulting linear systems are solved by frontal elimination.

The present algorithm employs a decoupled scheme, where the Navier-Stokes equations are solved first, in each timestep, lagging the temperature in the forcing term. Then the temperature is calculated, with the velocities as input. The time step is chosen adaptively using a PID control scheme based on controlling accuracy as determined by local truncation error estimates. Rayleigh-Benard and Marangoni flow calculations are presented, and the efficiency of the time step scheme is examined and compared with another time-stepping strategy [2].

Some promising results with PID control for timestep solution, such as, smooth variation in time step suggests that a robust algorithm is possible. With the PID control strategy we find approximate solutions with a smaller number of steps without any significant loss of accuracy. Future numerical experiments involve the calculation of the kinetic energy to improve time step selection. The nondimensional kinetic energy is a suitable parameter for monitoring the behavior of the liquid phase and for constructing bifurcation diagrams when time progress.

REFERENCES

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