3ª Prova Cálculo Numérico 09/1

Note Title 7/7/2009

1) a) pontos 1, 1.3, 1.8, 2

$$P_3(x) = 0.2624\,L_1(x) + 0.5878\,L_2(x) + 0.6931\,L_3(x)$$

$$L_1(x) = \frac{(x-1)(x-1.8)(x-2)}{(1.3-1)(1.3-1.8)(1.3-2)}$$

$$L_2(x) = \frac{(x-1)(x-1.3)(x-2)}{(1.8-1)(1.8-1.3)(1.8-2)}$$

$$L_3(x) = \frac{(x-1)(x-1.3)(x-1.8)}{(2-1)(2-1.3)(2-1.8)}$$

pontos 1, 1.3, 1.8

$$6 = \begin{vmatrix} 1.5-1 & 1-1.3 & 1-1.8 \\ 1.3-1 & 1.5-1.3 & 1.3-1.8 \\ 1.8-1 & 1.8-1.3 & 1.5-1.8 \end{vmatrix} = \begin{vmatrix} 0.5 & -0.3 & -0.8 \\ 0.3 & 0.2 & -0.5 \\ 0.8 & 0.5 & -0.3 \end{vmatrix}$$

$$P_2(1.5) = -0.03\left[\frac{0.2624}{-0.03} + \frac{0.5878}{-0.12}\right] = 0.4094$$

Condições:
$$\begin{cases} P_2(1) = 0 \\ P_2(1.3) = 0.2624 \\ P_2(1.8) = 0.5878 \end{cases}$$

os pontos 1.3, 1.8, 2

$$G = \begin{vmatrix} 1.5-1.3 & 1.3-1.8 & 1.3-2 \\ 1.8-1.3 & 1.5-1.8 & 1.8-2 \\ 2-1.3 & 2-1.8 & 1.5-2 \end{vmatrix} = \begin{vmatrix} 0.2 & -0.5 & -0.7 \\ 0.5 & -0.3 & -0.2 \\ 0.7 & 0.2 & -0.5 \end{vmatrix}$$

$$P_2(1.5) = 0.03\left[\frac{0.2624}{0.07} + \frac{0.5878}{0.03} + \frac{0.6931}{-0.07}\right]$$

$$= 0.4032$$

Condição de interpolação

$$\begin{cases} p_2(1.3) = 0.2624 \\ p_2(1.8) = 0.5878 \\ p_2(2) = 0.6931 \end{cases}$$

b) $\ln(1.9) = ?$ grau 3

pontos: 1.3, 1.8, 2, 2.2

Se for estimar o erro usando a expressão abaixo, não precisa fazer a tabela com todos os pontos

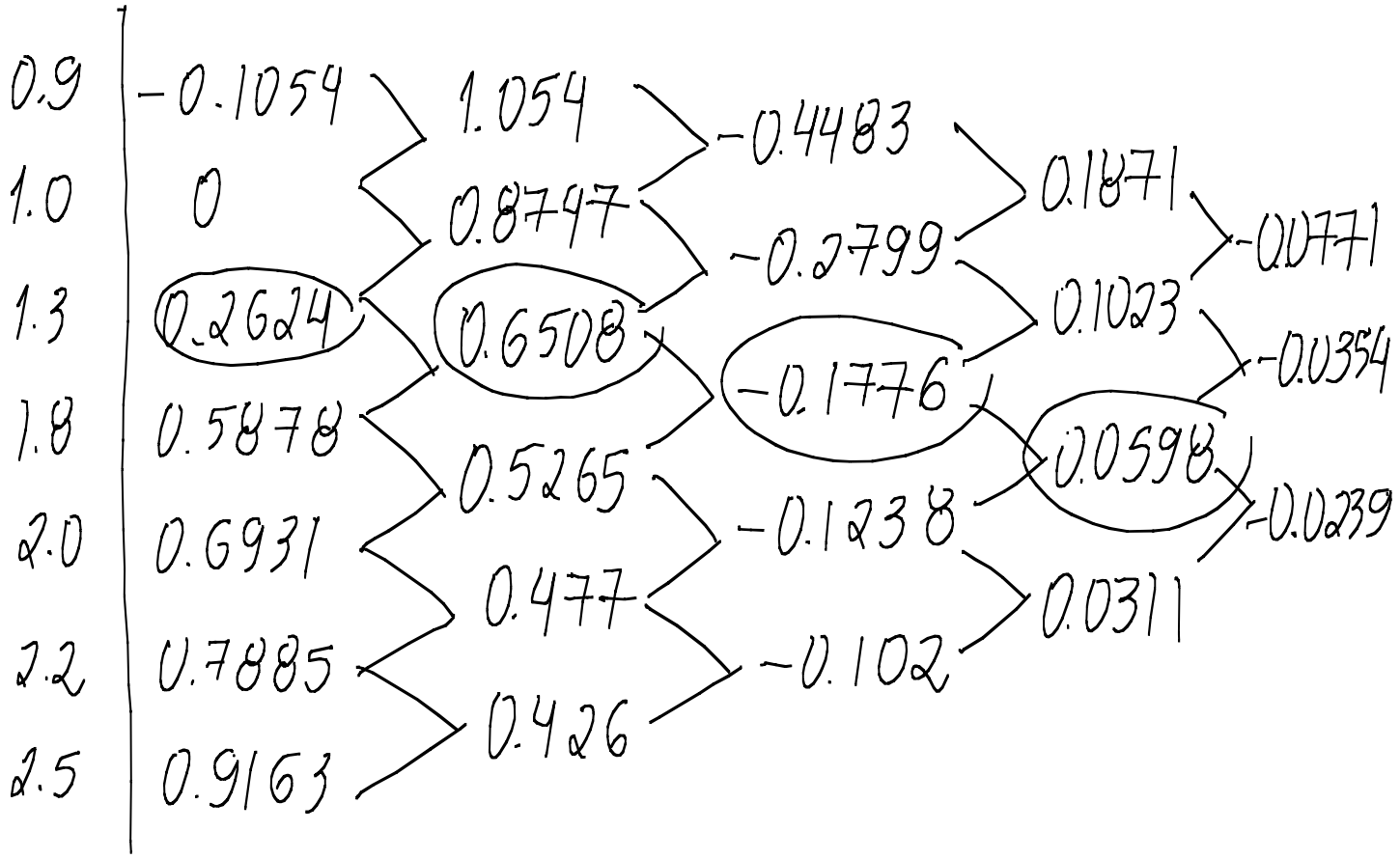
$$|erro| \leq \frac{M_4}{4!}\left|(1.9-1.3)(1.9-1.8)(1.9-2)(1.9-2.2)\right|$$

$$M_4 = \max_{x \in [1.3, 2.2]} |f^{IV}(x)|$$

$f^{I} = x^{-1}$ $\quad f^{III} = 2x^{-3}$

$f^{II} = -x^{-2}$ $\quad f^{IV} = 6x^{-4}$

$$M_4 = 6(1.3)^{-4} = 0.000158$$



$$P_3(1.9) = 0.2624 + 0.6508(1.9-1.3) -$$
$$-0.1776(1.9-1.3)(1.9-1.8) +$$
$$+0.0598(1.9-1.3)(1.9-1.8)(1.9-2)$$
$$= 0.6419$$

$$\text{erro exato} = 4.611 \times 10^{-5}$$

$$\text{erro} \approx 0.0771\,|(1.9-1.3)(1.9-1.8)(1.9-2)(1.9-2.2)|$$
$$= 0.000139$$

2) a) pontos 1, 1.4, 1.8, 2.2, 2.6

$$I_T = \frac{0.4}{2}\left[f(1.0)+f(2.6)+2\left[f(1.4)+f(1.8)+f(2.2)\right]\right] = 0.5901$$

b) pontos 1, 1.2, 1.4, 1.6, 1.8, 2, 2.2, 2.4, 2.6

$$I_S = \frac{0.2}{3}\left[f(1)+f(2.6)+2\left[f(1.4)+f(1.8)+f(2.2)\right]+4\left[f(1.2)+f(1.6)+f(2)+f(2.4)\right]\right]$$

$$= 0.5878$$

c)

pontos	0	0.88888	(t_0, A_0)
pesos	± 0.77459	0.55555	$(t_1, t_2, A_1 = A_2)$

$a = 0$ $\quad x = 0.75 + 0.75t$

$b = 1.5$ $\quad dx = 0.75\,dt$

$$\int_0^{1.5} e^{x^2} dx = \int_{-1}^{1} e^{0.5625(1+t)^2}\, 0.75\, dt$$

$$f(t) = e^{0.5625(1+t)^2}$$

$$\int_0^{1.5} e^{x^2} dx \approx$$

$$\approx 0.75 \left[A_0 f(t_0) + A_1 f(t_1) + A_2 f(t_2) \right]$$

$$= 4.048338$$

3 pontos $\Rightarrow$ integra sem erros pol grau ≤ 5

2 pontos $\Rightarrow$ " " " " " ≤ 3

$\Rightarrow$ precisa de 3 pontos

$$\int_0^1 (x^4+1)\, dx = \frac{1}{2} \int_1^1 \left[\left(\frac{1}{2} + \frac{1}{2} t \right)^4 + 1 \right] dt$$

$a = 0$
$b = 1$

$$x = \frac{1}{2} + \frac{1}{2} t$$

$$dx = \frac{1}{2} dt$$

$$= \frac{1}{2} \left[A_0 f(t_0) + A_1 f(t_1) + A_2 f(t_2) \right]$$