UniversidadeFederal do Espírito Santo
Departamento de Informática

Armazenamento de Matrizes Esparsas

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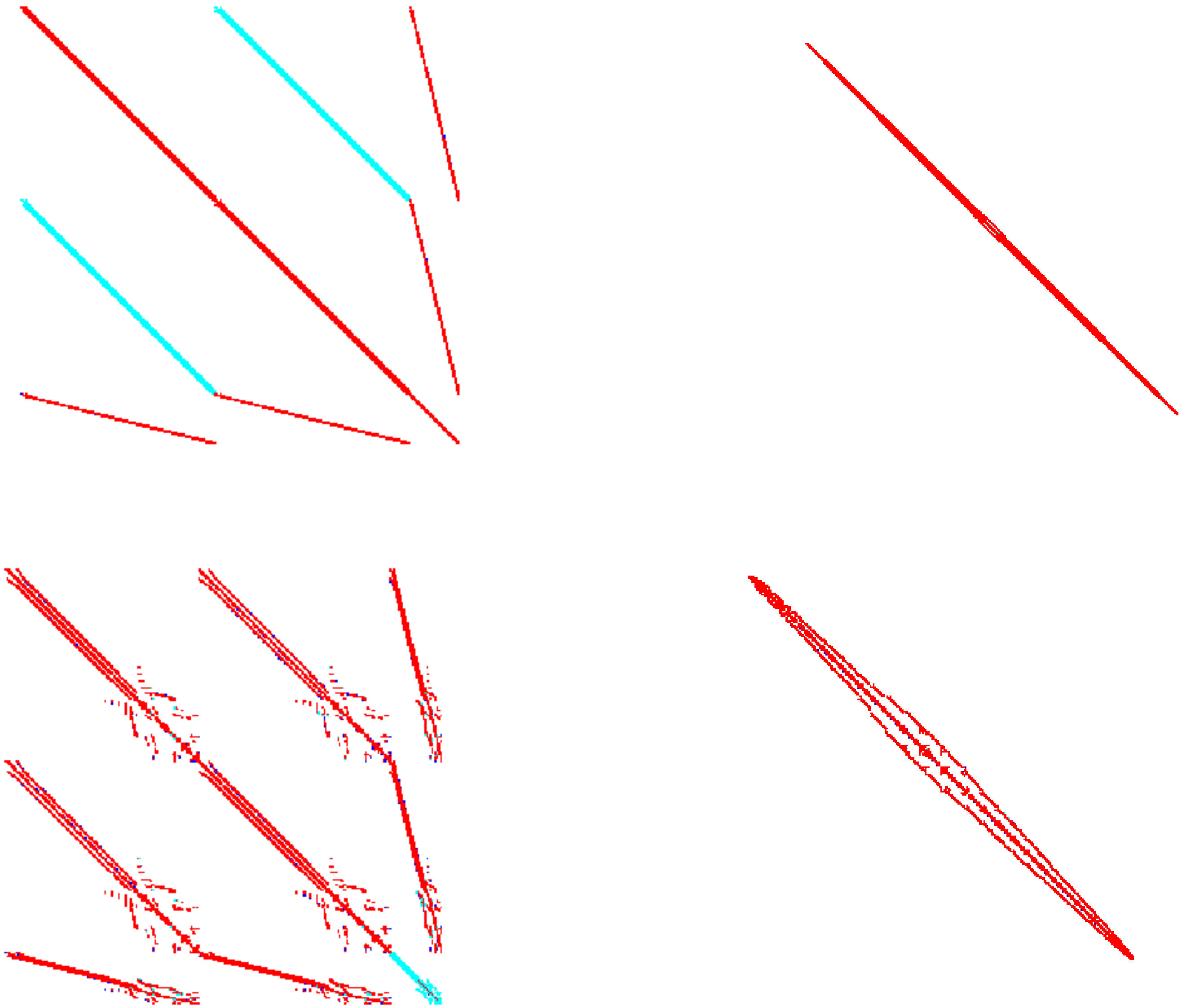
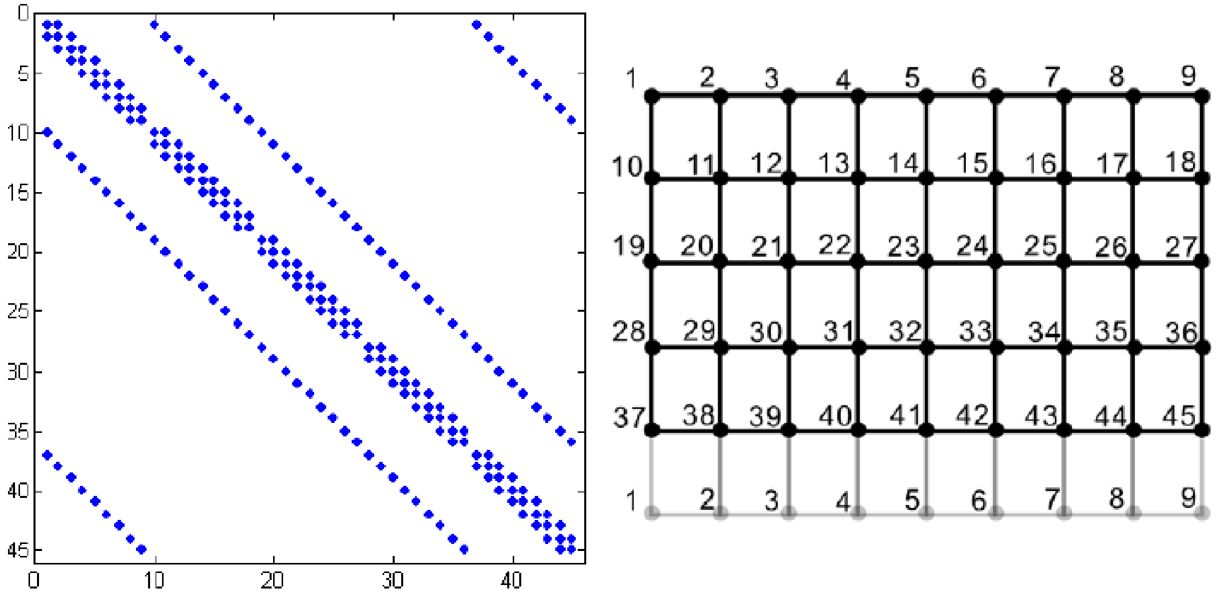
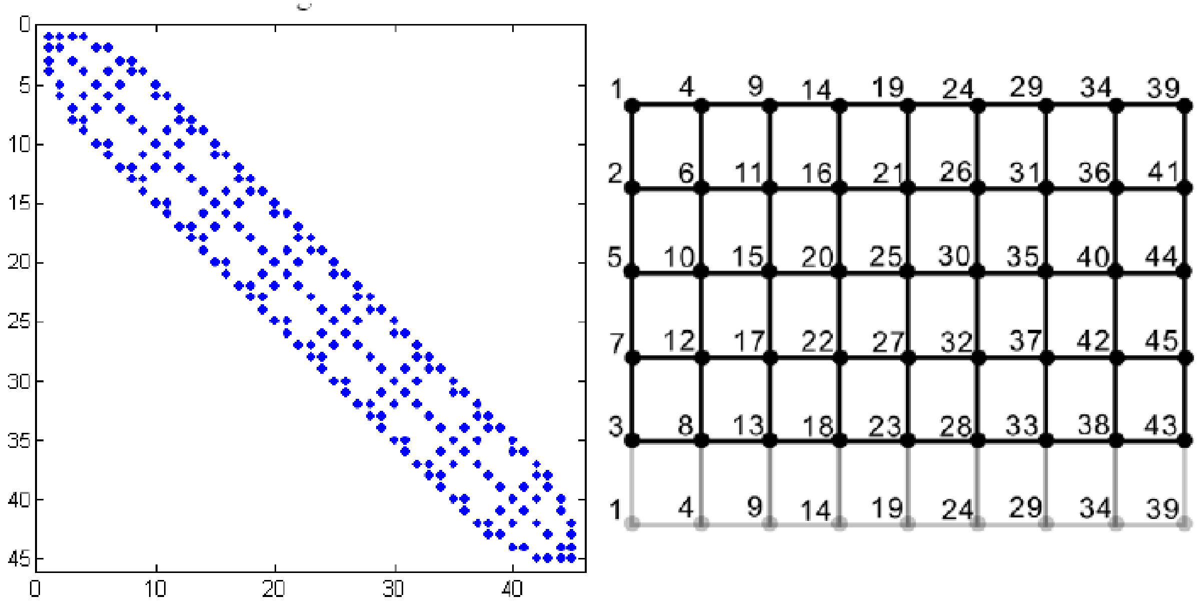
Matrizes Esparsas

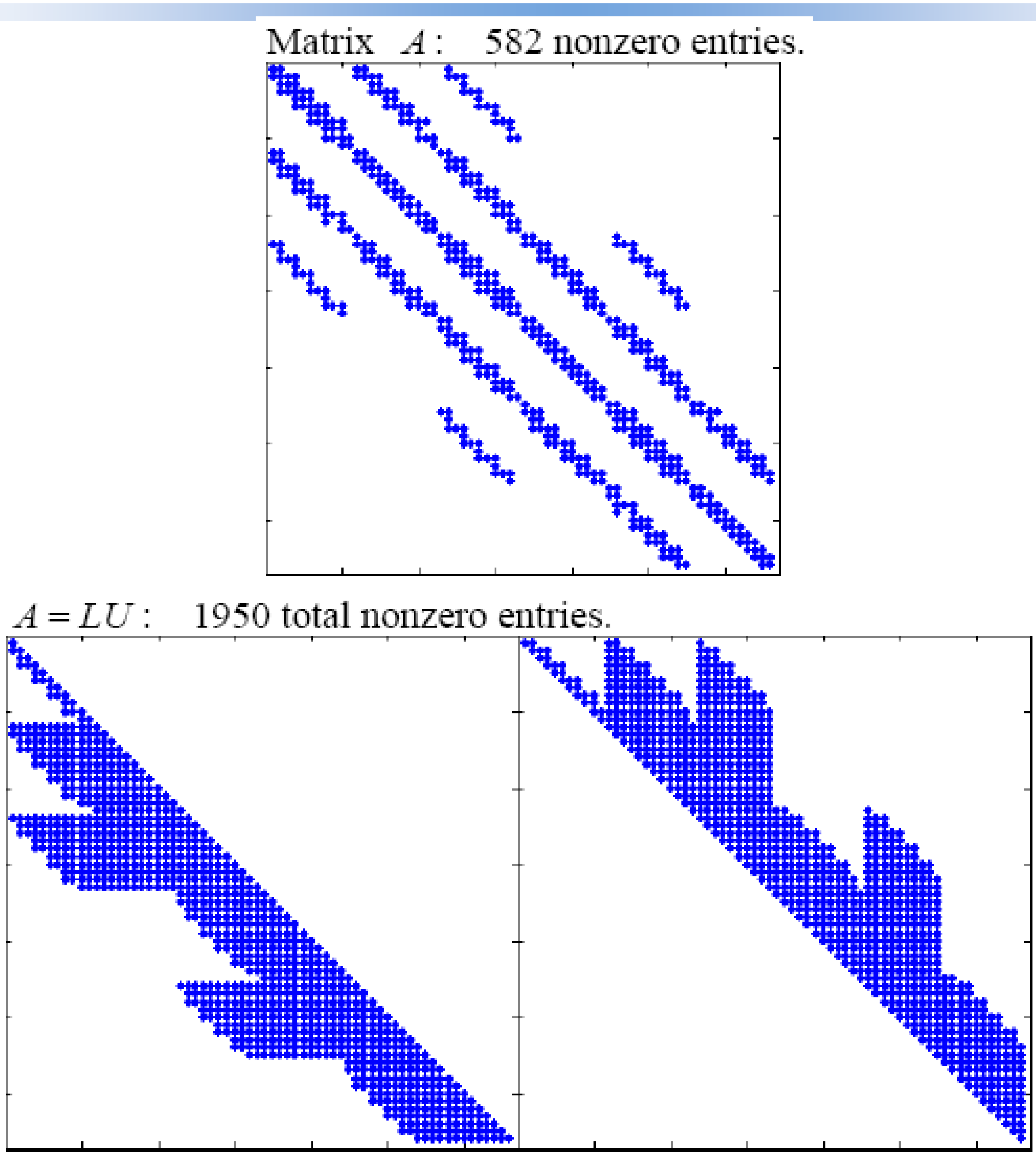


Matrizes Esparsas

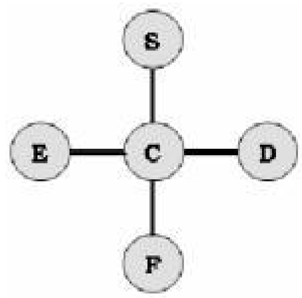


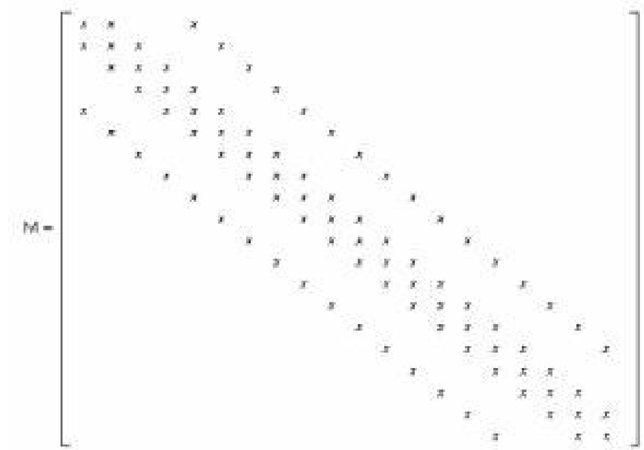


Matrizes Esparsas



Formato Diagonal para Matrizes pentadiagonais





$$A=\begin{bmatrix}
4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-1 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 4 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 4
\end{bmatrix}
\Rightarrow M=\begin{bmatrix}
0 & 0 & 4 & -1 & -1\\
0 & -1 & 4 & -1 & -1\\
0 & -1 & 4 & -1 & -1\\
0 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & -1\\
-1 & -1 & 4 & -1 & 0\\
-1 & -1 & 4 & -1 & 0\\
-1 & -1 & 4 & -1 & 0\\
-1 & -1 & 4 & 0 & 0
\end{bmatrix}
\rightarrow Dif=\begin{bmatrix}-4 & -1 & 0 & 1 & 4\end{bmatrix}$$

Formato CSR para Matrizes Esparsas
(Compress Sparse Row)

$$A = \begin{array}{|c|c|c|c|c|c|} \hline a_{11} & & a_{13} & & & \\ \hline & a_{22} & & & a_{25} & \\ \hline & & a_{33} & a_{34} & & \\ \hline a_{41} & & & & a_{45} & \\ \hline & & a_{53} & a_{54} & & \\ \hline \end{array} \Rightarrow$$

$$AA = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline a_{11} & a_{13} & a_{22} & a_{25} & a_{33} & a_{34} & a_{41} & a_{45} & a_{53} & a_{54} \\ \hline \end{array}$$

$$JA = \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline 1 & 3 & 2 & 5 & 3 & 4 & 1 & 5 & 3 & 4 \\ \hline \end{array}$$

$$IA = \begin{array}{|c|c|c|c|c|} \hline 1 & 3 & 5 & 7 & 9 \\ \hline \end{array}$$

Nnz = número de coeficientes não nulos

Formato CSR
(Compress Sparse Row)

$$A = \begin{pmatrix} 10 & 0 & 0 & 0 & -2 & 0 \\ 3 & 9 & 0 & 0 & 0 & 3 \\ 0 & 7 & 8 & 7 & 0 & 0 \\ 3 & 0 & 8 & 7 & 5 & 0 \\ 0 & 8 & 0 & 9 & 9 & 13 \\ 0 & 4 & 0 & 0 & 2 & -1 \end{pmatrix}$$

val	10	-2	3	9	3	7	8	7	3 ··· 9	13	4	2	-1
col_ind	1	5	1	2	6	2	3	4	1 ··· 5	6	2	5	6

row_ptr	1	3	6	9	13	17	20

- n - ordem de *A*
- nnz - número de coeficientes não nulos
- 2nnz + n+1 - número de alocações para armazenar *A*
- val(k) = a(i,j), col_ind(k) = j, row_ptr(i) <= k < row_ptr(i+1)

Formato CDS (Armazena matriz Banda)
(Compressed Diagonal Storage)

$$A = \begin{pmatrix} 10 & -3 & 0 & 0 & 0 & 0 \\ 3 & 9 & 6 & 0 & 0 & 0 \\ 0 & 7 & 8 & 7 & 0 & 0 \\ 0 & 0 & 8 & 7 & 5 & 0 \\ 0 & 0 & 0 & 9 & 9 & 13 \\ 0 & 0 & 0 & 0 & 2 & -1 \end{pmatrix}$$

val(:,-1)	0	3	7	8	9	2
val(:, 0)	10	9	8	7	9	-1
val(:,+1)	-3	6	7	5	13	0

$$A = \begin{pmatrix} 10 & -3 & 0 & 1 & 0 & 0 \\ 0 & 9 & 6 & 0 & -2 & 0 \\ 3 & 0 & 8 & 7 & 0 & 0 \\ 0 & 6 & 0 & 7 & 5 & 4 \\ 0 & 0 & 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 0 & 5 & -1 \end{pmatrix}$$

val(:,-1)	0	0	3	6	0	5
val(:, 0)	10	9	8	7	9	-1
val(:,+1)	0	-3	6	7	5	13
val(:,+2)	0	1	-2	0	4	0

- n - ordem de *A*
- p - número de diagonais não-nulas abaixo da diagonal
- q – número de diagonais não-nulas acima da diagonal

Formato JDS (Armazena matriz Banda)
(Jagged Diagonal Storage)

$$\begin{pmatrix} 10 & -3 & 0 & 1 & 0 & 0 \\ 0 & 9 & 6 & 0 & -2 & 0 \\ 3 & 0 & 8 & 7 & 0 & 0 \\ 0 & 6 & 0 & 7 & 5 & 4 \\ 0 & 0 & 0 & 0 & 9 & 13 \\ 0 & 0 & 0 & 0 & 5 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 10 & -3 & 1 & \\ 9 & 6 & -2 & \\ 3 & 8 & 7 & \\ 6 & 7 & 5 & 4 \\ 9 & 13 & & \\ 5 & -1 & & \end{pmatrix}$$

val(:,1)	10	9	3	6	9	5
val(:,2)	−3	6	8	7	13	−1
val(:,3)	1	−2	7	5	0	0
val(:,4)	0	0	0	4	0	0

col_ind(:,1)	1	2	1	2	5	5
col_ind(:,2)	2	3	3	4	6	6
col_ind(:,3)	4	5	4	5	0	0
col_ind(:,4)	0	0	0	6	0	0

- n - ordem de *A*
- p - número de diagonais não-nulas abaixo da diagonal
- q – número de diagonais não-nulas acima da diagonal

Formato SKL (Armazena Matriz Skyline)
(Armazenamento Skyline)

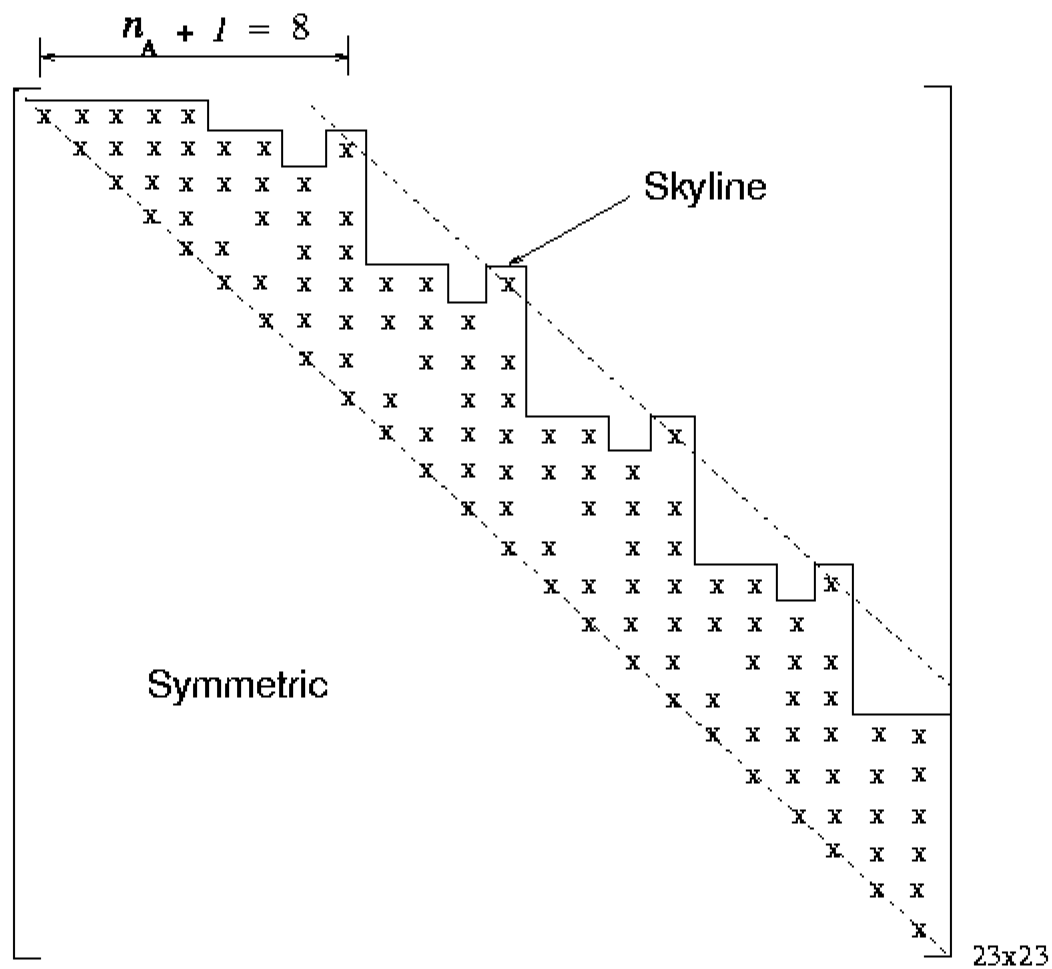
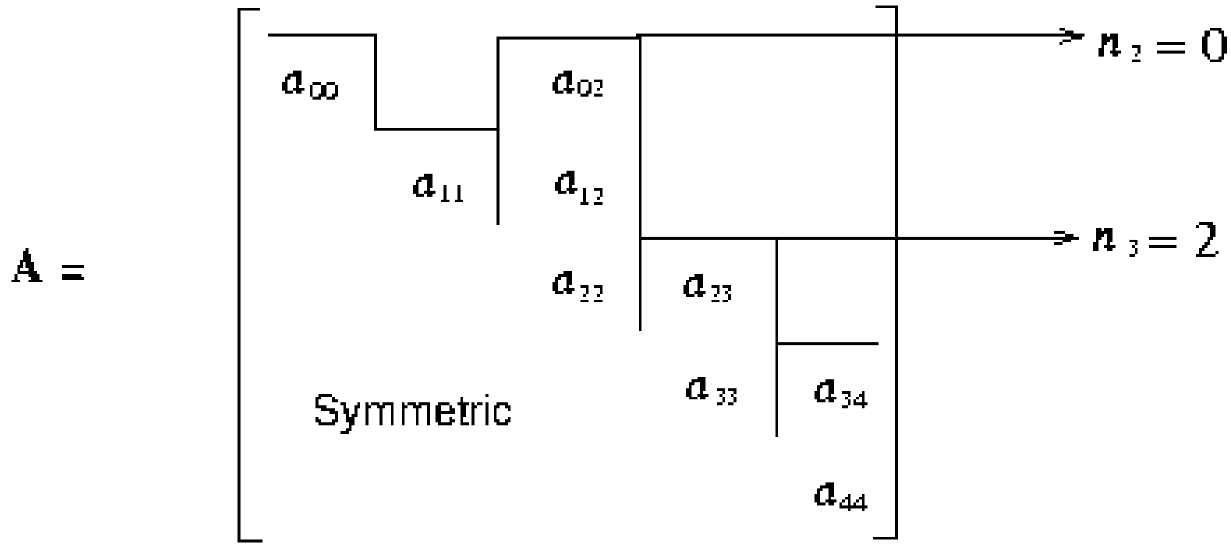


Figure 4.4: The structure of the system matrix $\mathbf{A}^t$ of our physics-based fish model, the skyline of $\mathbf{A}^t$ and its half-bandwidth $n_{\mathbf{A}}$. Each "x" represents a non-zero entries.

Formato SKL (Armazena Matriz Skyline)
(Armazenamento Skyline)



$$\begin{array}{cccccccccc} & \bar{a}_0 & \bar{a}_1 & \bar{a}_2 & \bar{a}_3 & \bar{a}_4 & \bar{a}_5 & \bar{a}_6 & \bar{a}_7 & \bar{a}_8 \end{array}$$

$$\bar{A} = \begin{bmatrix} a_{00} & a_{11} & a_{22} & a_{12} & a_{02} & a_{33} & a_{23} & a_{44} & a_{34} \end{bmatrix}$$

$$MAX\bar{A} = \begin{bmatrix} 0 & 1 & 2 & 5 & 7 \end{bmatrix}$$

Figure 4.5: An example of the skyline storage scheme. A is the example system matrix; $\bar{A}$ is the storage array and $MAX\bar{A}$ is the index array.